Hedging issues for CDOs

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April, 6, 2008

Abstract

This paper is a primer on the hedging and the risk management of CDO tranches. It intends to provide a global perspective on the current issues and refers to research papers for modelling and mathematical details. Though the basics of the risk management within the Gaussian copula model are not discussed, we review some issues which may eventually lead to the decline of the current market approach. We consider the computation of spread sensitivities in arbitrage-free dynamic models based on affine intensities and on Markov chains. We show that the one dimensional Markov chain for the aggregate loss process may viewed as the analogue for credit markets of the local volatility model in equity derivatives markets. In such a framework, CDO tranches can be fully replicated with the credit default swap index and a risk-free asset and the corresponding deltas correspond to a "sticky implied tree model". Hedging issues related to tranchelets on standard indices or to CDO tranches on bespoke portfolios are dealt with. We also investigate some paths for the future, including the use of credit default swaps of different maturities to cope simultaneously with default and credit spread risks, local and asymptotic hedging approaches.

Keywords: CDOs, hedging, replication strategies, market completeness, Markovian contagion models, quadratic hedging

JEL: G13

Introduction

The risk management and the hedging of CDOs and related products are topics of tremendous importance, especially given the 2007 credit turmoil. The risks at hand are usually split into different categories, which may sometimes overlap, such as credit spread and default risks, correlation and contagion risks. These will be the centre of our discussion. The 2007 credit crisis also drove attention to counterparty risk and related issues such as collateral management, downgrading of guarantors and of course liquidity issues. For simplicity, these will not be dealt within this paper[3].

Though this contribution is non mathematical in nature, we will comment about technical issues and refer to the corresponding research papers for the readers with more quantitative tendencies. We will introduce the main hedging issues and a state of the art of theoretical ideas, given that the techniques are quickly evolving and have not yet come to maturity. Let us also stress that the credit derivatives modelling area is characterized by a number of competing approaches with pros and cons, none of them could pretend to be the ultimate standard.

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[3] See Gibson (2007) for a discussion of the issues involved.

Surprisingly enough, since pricing at the cost of the hedge is the cornerstone of the derivatives modelling field, models that actually connect pricing and hedging issues have been studied after the one factor Gaussian copula model became a pricing standard. This discrepancy with the equity or interest derivatives fields can actually be seen as a weakness and one can reasonably think that further researches in the credit area will aim at closing the gap between pricing and hedging.

Before proceeding further, let us recall the main features in a hedging and risk management problem, which come to light whatever the underlying risks:

- A first issue is related to the choice and the liquidity of the hedging instruments: typically, one could think of credit index default swaps, CDS on names with possibly different maturities, standardized synthetic single tranche CDOs and even other products such as equity put options, though this will not be detailed in this paper. We reckon that the use of equity products to mitigate risks can be useful in the high yield market, but this is seemingly not the case for CDO tranches related to investment grade portfolios.
- A second issue is related to the products to be hedged. In the remainder, we will focus on basket credit derivatives, such as First to Default Swaps, CDO tranches, bespoke CDOs or tranchelets. We will leave aside interest rate or foreign exchange hybrid products, credit spread options and exotic basket derivatives such as leveraged tranches, forward starting CDOs or tranche options.
- A third issue relies on the choice of the hedging method. The mainstream theoretical approach in mathematical finance favours the notion of replication of complex products through dynamic hedging strategies based on plain underlying instruments. However, it is clear that in many cases, risk can be mitigated by offsetting long and short positions, providing either a complete clearing or more usually leaving the dealer exposed to some basis albeit small risk. Moreover, such an approach is obviously quite robust to model risk. Unfortunately, there are some imbalances in customer demand and investment banks can be left with rather large outstanding positions on parts of the capital structure that must be managed up to maturity.

Since some risks within CDO tranches cannot be reasonably fully hedged, a suitable reserve policy is usually required. This applies obviously to model risk, especially for some illiquid bespoke tranches and to default risk, notably when risk management is primarily concerned with shifts in credit spreads.

The paper is organised as follows. Section I focuses on the computation of sensitivities with respect to market or risk parameters given a pricing model for CDO tranches. This approach is primarily used by market participants, as is also the case in other fields of the derivatives industry. We however emphasize that the mainstream pricing model for CDO tranches is neither exempt from static arbitrage opportunities nor related to the cost of the hedge of a self-financed replicating strategy. Section II also deals with hedging CDO tranches through sensitivity analysis but the focus is put on models that are calibrated to liquid tranche quotes. These standardized tranches can be seen as hedging instruments that should perform better than plain credit default swaps with respect to correlation and model risk. Section III is devoted to the hedging of CDO tranches in a complete market framework. By completeness, we mean that CDO tranches can be replicated by a dynamic strategy based on traded instruments such as single name credit default swaps, at least within the model assumptions. Though this approach encompasses copula and Markov chain models, these differ regarding implementation of hedging strategies. Eventually, Section IV briefly discusses other approaches that will be a field for future research.

I) Risk management of CDO tranches based on sensitivity approaches

In these approaches the pricing engine is given and there is no theoretical connexion between the price of the tranche and the global cost of the hedging strategy. We may even say that in most examples discussed below, there does not exist a replicating strategy.

For example, on the industry side, the market participants rely on the one factor Gaussian copula model amended with a base correlation structure. The principles of the hedging and risk management are fairly simple: the pricing tools are used to compute sensitivities to market inputs and to market parameters, such as credit spreads of the constituents of the reference credit portfolio. The main focus is put on credit spread risk, while default risk is usually dealt with a reserve policy. Such risks are managed thanks to credit index default swaps or CDS on the underlying names of the basket. Other risks, such as idiosyncratic and parallel gamma[4] credit spread risks, or correlation exposure can be in principle managed by trading liquid index tranches across the capital structure. Petrelli, Zhang, Jobst and Kapoor (2007) conduct a thorough study of popular trading strategies involving CDO tranches hedged with respect to small shifts in credit spreads. This shows that convexity and correlation risks may actually lead to a poor hedging performance of such trades.

Let us emphasize a key issue when computing credit deltas in the one factor Gaussian copula model with base correlations. There are actually two approaches that can be denoted as “sticky strike” and “sticky delta” to parallel the terminology used in equity derivatives markets (see Derman (1999)). In the sticky strike approach, the base correlations are kept unchanged when bumping the credit curves. When computing “sticky deltas”, one takes into account the change in base correlations due to the change in the moneyness of the tranche when credit spreads move up: the equity tranche becomes more junior, which actually leads to using a smaller base correlation. In other words, in the sticky delta approach an increase in credit spreads is associated with a smaller dependence between default events. As a consequence, the sticky delta of an equity tranche is lower than the delta computed under the sticky strike approach.

Morgan and Mortensen (2007) have investigated some anomalies when using the base correlation for the computation of sensitivities. They show that credit spread deltas on

[4] Idiosyncratic Gamma is also denoted as iGamma and referred to as “microconvexity”. Parallel Gamma is also known as Index Gamma and referred to as “macroconvexity”.

iTraxx S7 5Y [12-22%] tranche can be negative due to the steepness of the base correlation curve. Such a counterintuitive effect[5] is illustrated in the graph below:

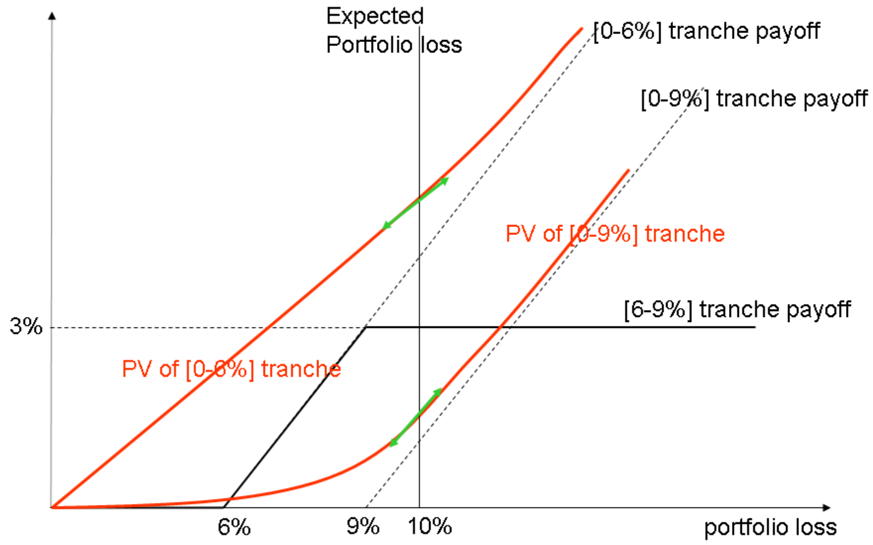

Figure 1. Negative deltas due to the steepness of the base correlation curve.

Example: consider a mezzanine [6-9%] tranche on a bespoke portfolio. The expected loss on the portfolio at inception is equal to 10%. Figures in red show the present value of the default leg of the [0-6%] and [0-9%] base correlation tranches as a function of the expected loss on the portfolio. The base correlation for the [0-9%] is assumed to be very high, thus the volatility of the reference portfolio is quite small as the time value of the base tranche. Conversely, the base correlation of the [0-6%] tranche is much smaller which is consistent with steep upward base correlation curves. Thus, the volatility of the (same) reference portfolio is much higher as the time value of the option. The present value of the default leg of the mezzanine [6-9%] tranche is the difference between the present values of the [0-6%] and [0-9%] base correlation tranches and should remain between 0 and 3% to avoid plain arbitrage opportunities. Given this constraint, it may be (see Figure 1) that the delta of the more junior tranche is smaller than the delta of the more senior tranche for some levels of expected portfolio loss. In such regions, the present value of the mezzanine tranche will decrease as the expected loss on the underlying portfolio increases which is rather unlikely.

We refer to Jobst (2007) or Meissner, Hector and Rasmussen (2008) for further discussions about hedging CDOs within the Gaussian copula framework. On the numerical side, Andersen, Sidenius and Basu (2003), Gregory and Laurent (2003), Iscoe and Kreinin (2006) provide semi-analytical techniques to compute sensitivities within the Gaussian copula framework. Brasch (2004), Joshi and Kainth (2004), Rott and Fries (2005), Chen and Glasserman (2006) detail some improvements of the Monte Carlo approach which are applicable to the pricing and hedging of CDO tranches, especially when one falls outside the factor framework.

The sensitivity approach also applies to other copulas that, contrary to the base correlation approach provide arbitrage-free CDO tranche quotes. Schloegl, Mortensen and Morgan (2008) prove that tranchelet sensitivities are always positive in such a

[5] Schloegl, Mortensen and Morgan (2008) show that such effects could occur in arbitrage-free models.

framework. However, some other issues come to light such as huge discrepancies between individual name deltas and subsequently very large positive or negative idiosyncratic Gammas in high correlation regimes as observed in 2008 on the iTraxx and CDX markets. This is illustrated for example in the stochastic correlation described by Burtschell, Gregory and Laurent (2007). The bumps in Figure 2 are related to the comonotonic state and the heterogeneity amongst credit spreads. Such a rather counterintuitive pattern precludes the use of the credit default swap index as hedging tool for CDO tranches. Other heterogeneity effects in individual credit deltas are reported by Houdain (2006).

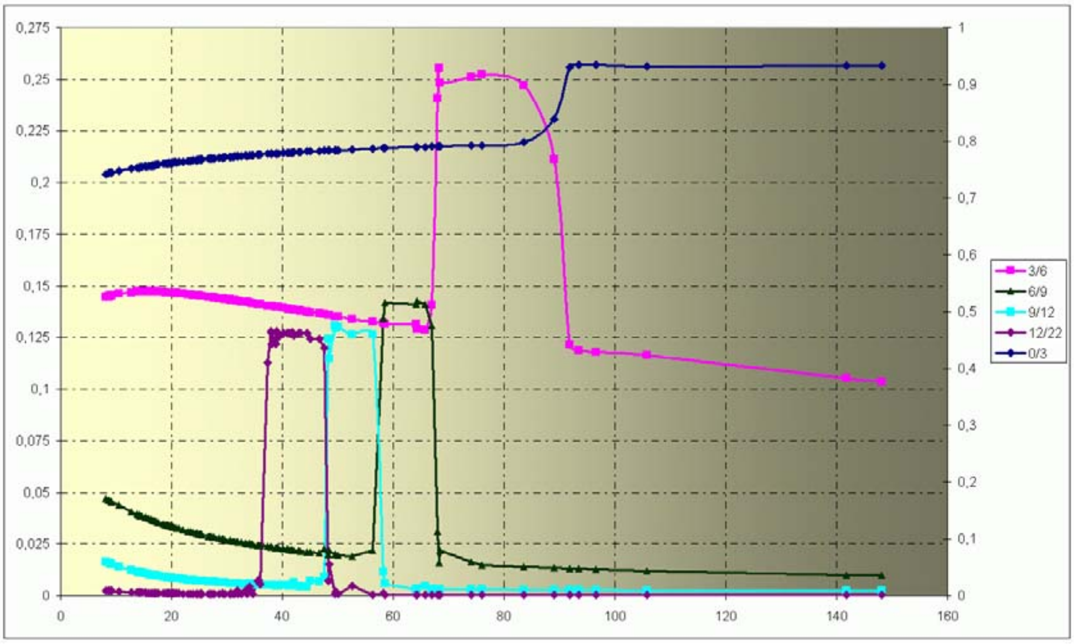

Figure 2. irregular patterns of individual names deltas in regimes of high correlation.

Figure 2 exhibits CDO tranche deltas with respect to the level of credit spreads computed on 31-August-2005. Nominal is equal to 125. 5 year credit spreads on the *x*-axis are expressed in basis points per annum. Credit deltas of the equity tranche are on right axis. Figure 2 shows that individual credit deltas may actually differ significantly from one name to another.

Though the pricing methodology differs, Eckner (2007), Arnsdorf and Halperin (2007) provide some examples of the use of dynamic arbitrage free pricing models for the computation of sensitivities to credit spreads and thus of hedge ratios with respect to credit default swaps.

Eckner (2007) model relies on an affine specification of default intensities. Conditionally on the path of default intensities, default times are independent (i.e. there are no contagion effects at default times). The model is parametric with respect to the term structure of credit spreads and to CDO tranches. Calibration of the model parameters to credit spreads and liquid tranche quotes on the CDX NA IG5 index in December 2005 is provided and hedge ratios with respect to the credit default swap index are then computed[6]. The sensitivities of CDO tranche and index prices are computed with respect to a uniform and relative shift of individual intensities. The approach can be extended in order to compute different hedge ratios with respect to the single name default swaps. However, the overall procedure, including the calibration and the computation of individual hedge ratios is likely to be rather involved. The model deltas can be compared with those computed from the Gaussian copula model. As can be seen from Table 1, though the figures differ, the orders of magnitude are roughly the same. The equity tranche deltas computed in Eckner (2007) are slightly larger than those computed under the Gaussian copula, as in a "sticky delta" approach. Such a result is consistent with a market where an increase in the average credit spread is the outcome of some idiosyncratic shifts and an increase in the dispersion of credit spreads. This is typical of the correlation crisis in May 2005, which was actually associated with smaller correlations on the equity tranches.

Tranches	[0-3%]	[3-7%]	[7-10%]	[10-15%]	[15-30%]
market deltas	18.5	5.5	1.5	0.8	0.4
model deltas	21.7	6.0	1.1	0.4	0.1

Table 1. Market deltas and model deltas as in Eckner (2007).

Arnsdorf and Halperin (2007) consider a Markov chain that accounts for the dynamics of defaults and credit spreads. This can be seen as a "two dimensional" Markov chain. Contrary to the previous model, defaults are informative and credit spreads jump at the arrival of defaults. The theoretical properties of the model with respect to completeness are not studied but Arnsdorf and Halperin (2007) compute deltas of standard iTrraxx tranches with respect to the corresponding credit default swap index. As in Eckner (2007), the deltas with respect to individual credit default swaps are not provided. However, one could think of using the random thinning procedure discussed in Giesecke and Goldberg (2005) or Giesecke (2008) to provide such individual deltas.

Tranches	[0-3%]	[3-6%]	[6-9%]	[9-12%]	[12-22%]
market deltas	26.5	4.5	1.3	0.7	0.3
model deltas	21.9	4.8	1.6	0.8	0.4

Table 2. Market and model deltas as in Arnsdorf and Halperin (2007).

Table 2 shows some market (computed under the Gaussian copula model) and model deltas (corresponding to "model B" in Arnsdorf and Halperin (2007)) in March 2007, for five year CDO tranches. As in Table 1, it can be seen that the figures are roughly the same. However, it is noticeable that equity tranche deltas are smaller when using the Markov chain.

Though the previous contributions were focused on sensitivities with respect to credit spreads, it is straightforward to extend the approach to other risk parameters such as correlation sensitivities. Figure 3 taken from Gregory and Laurent (2004) shows how

[6] Let us remark that this model would hardly be calibrated during the 2008 crisis on CDX and iTraxx tranches.

correlation sensitivities do depend on the riskiness of the names within the underlying portfolio. For instance, the correlation sensitivity of equity tranche premiums is much higher when considering names with higher spreads.

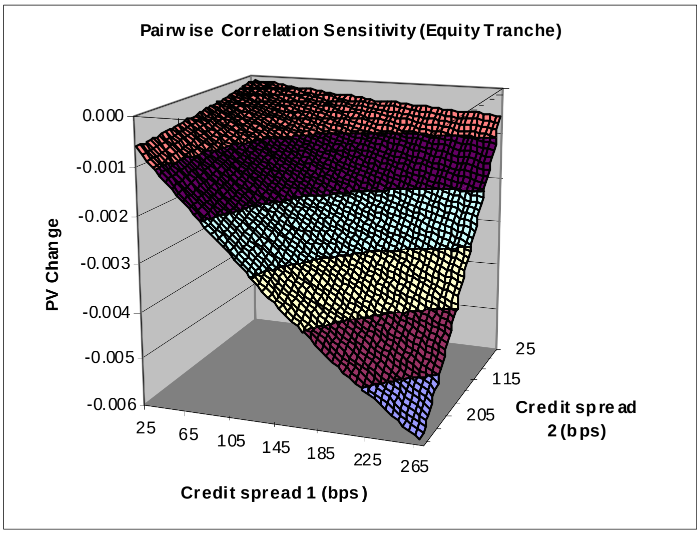

Figure 3. Correlation sensitivities with respect to the level of credit spreads. Changes in the present value of the equity tranche are on the z-axis. Spreads of the pairs of names considered are on the x and y axes.

A clear issue is that the market pricing tools do not derive from the standard approach in mathematical finance, where the price of a derivatives contract is the cost of a fully replicating self-financing strategy. As far as the investors or investment banks deal with rather illiquid contracts, that are rather marked-to-model than marked-to-market, internal inconsistencies between the pricing and hedging approaches will have only small effects on the dynamics of the P&L. However, it may be that discrepancies gradually arise through time, such as an unexplained time decay effects, which is clearly a problem for dealers who need to manage contracts up to lengthy maturities.

II) Hedging CDO tranches with liquid CDO tranches

Since there is now a liquid market for standardized tranches on iTraxx or CDX indexes, it is then quite natural to use these tranches as hedging tools. Clearly, this assumes that the market for standardized tranches is not segmented which might be the case in periods of turmoil that market participants have already experimented. Petrelli, Zhang, Jobst and Kapoor (2007) conducted a review of popular positive carry hedging strategies, involving several CDO tranches and showed some rather unexpected outcomes, due to the complexity of the dynamics of the capital structure. Such unpleasant issues are also discussed in Patel (2007).

We will firstly discuss the hedging of tranches with non standard attachment or detachment points on standard indices and in a second step the more involved issue of hedging CDO tranches on bespoke portfolios.

II.1) Hedging of CDO tranches on standard indices

Walker (2005) provides a detailed account of the price constraints of CDO tranches. Thanks to linear programming techniques, he derives maximum and minimum prices of CDO tranches consistent with the absence of arbitrage opportunities. An outcome of the approach is the "super-replication" of non standard tranches or of tranchelets based on portfolios of liquid tranches. This is a robust model-free approach to the pricing and hedging. However, in some cases the range of arbitrage-free prices can be large and a model-based approach may be required.

Whatever the chosen model, it can be seen as an interpolation or extrapolation pricing tool depending on the attachment or detachment points. Since the model is usually calibrated to liquid tranche quotes, it is possible to compute sensitivities with respect to these input quotes (including the credit default swap index) and thus deltas with respect to these liquid tranches.

One could think of using either parametric or non parametric approaches. For instance, Burtschell, Gregory and Laurent (2007) compare a stochastic correlation model and the random factor loading model of Andersen, Sidenius and Basu (2003). Though both models can be reasonably well calibrated to standardized iTraxx tranche quotes, they provide strikingly different prices for very junior tranches, say a [0-1%] tranche. Though the sensitivity of these prices to the liquid tranches quotes is not studied, it casts some doubt about the robustness of the corresponding hedge ratios. Similar issues appear at the other end of the capital structure. It is well known that prices of bespoke super-senior tranches (say a [30-100%] tranche on the iTraxx index) are quite sensitive to assumptions on recovery rates. As a consequence, extrapolating prices even with the help of an arbitrage-free model could be hazardous and this lack of robustness is likely to apply to hedge ratios too.

Amongst the non parametric approaches, one can think of purely smoothing procedures using liquid tranche quotes as inputs. When applied with caution, this can produce arbitrage-free tranche quotes. However, the resulting prices are quite sensitive to the smoothing method as shown in Gregory and Laurent (2008). Once again, this is not good news regarding the robustness of the resulting hedge ratios. The non parametric approach can be improved as in Hull and White (2006) who derive distributions of conditional default probabilities from market quotes. Another non parametric approach, based on the static loss-surface model is discussed in Walker (2008) who also provides a number of empirical results.

However, as can be seen from the above comments, more detailed and systematic studies of the robustness of the hedge ratios of bespoke tranches with respect to standardized tranches remain to be done.

II.2) Hedging of CDO tranches on bespoke portfolios

Though the hedging of CDO tranches on bespoke portfolios is quite topical, we are not aware of many published papers related to this subject.

We can mention Smithson and Pearson (2007) who investigate different mapping procedures for the pricing of such bespoke tranches. Unsurprisingly, they obtain a wide range of prices depending on the mapping procedure. However, it appears that most of the variation is due to the difference in the source of base correlations (iTraxx and CDX). Of course, it is not clear why an arbitrary scheme would provide the correct correlations between names of a bespoke portfolio whose constituents may differ a lot from standard indices. As discussed above, it is likely that the sensitivity of the prices of bespoke tranches with respect to liquid quotes inputs depends also a lot on the mapping procedure and significant model risk could well occur.

The mapping procedures actually lead to unexpected sensitivities: the present value of a tranche on a bespoke portfolio depends upon credit spreads on names which are in the somehow arbitrary reference portfolio and that may not appear in the bespoke portfolio. Such extraneous deltas cannot be cancelled out without introducing some discrepancies between the pricing and hedging procedures.

III) CDO pricing models associated with perfect hedging

There are a number of papers related to this approach such as Bielecki, Crépey, Jeanblanc and Rutkowski (2007), Bielecki, Jeanblanc and Rutkowski (2007b), Frey and Backhaus (2007a, 2007b), Laurent, Cousin and Fermanian (2007). These contributions are primarily concerned with the hedging of default events. Credit spread or correlation risks may actually occur but stem from the arrival of defaults. In this pricing and hedging approach, one can perfectly replicate a CDO tranche payoff thanks to a self-financed dynamic strategy based upon single name credit default swaps.

Laurent, Cousin and Fermanian (2007), Frey and Backhauss (2007a, 2007b) consider a continuous time Markov chain for the loss process. The number of states within the Markov chain is equal to $n+1$ where n is the number of names in the reference portfolio. This is associated with a simple modelling of pre-default name intensities, all these being equal and driven only by the number of defaults. Credit spreads of non defaulted names jump at default times, thus the model is associated with contagion effects. As far as the pricing is concerned the model is well-known and has been studied by Schönbucher (2006) or Herbertsson (2007) among others. The model can be perfectly calibrated to CDO tranche quotes as observed in the market. Thanks to a martingale representation theorem for point processes, it is possible to express the payoff of a CDO tranche as the outcome of a dynamic strategy based on the credit default swap index related to the underlying portfolio. The numerical implementation is fairly easy and can be conducted through a binomial tree on the loss process. Let us remark that the continuous time Markov chain can be seen as the analogue of the local volatility model of Dupire (1994): it is a one dimensional Markov complete model and there is a one to one correspondence between the dynamics of the "underlying" process (here the aggregate loss) and a complete set of CDO tranche quotes thanks to the forward Kolmogorov equations. The discrete binomial version of the model corresponds to the implied tree of Derman and Kani (1994).

Dealing with idiosyncratic risks such as the widening of a given credit spread within the reference portfolio is of great importance as far as risk management of equity tranches is concerned. This cannot be taken easily into account in a low dimensional Markovian contagion model since dealing with heterogeneity breaks down the Markov property[7] of the aggregate loss process. On theoretical grounds, one could in principle consider a Markov chain with 2^n states, but this is not applicable to iTraxx or CDX tranches due to obvious computational issues.

Though the Markovian contagion model is quite easy to understand and implement, provides a perfect fit to quoted CDO tranches and a tight link between the pricing and the hedging of tranches, it has another seemingly major drawback: credit spreads are only driven by the arrival of defaults. Meanwhile, credit spreads are constant and do not have specific dynamics as is obviously seen in the credit market. Moreover, credit spreads can usually (in case of positive dependence) only increase through time and do no mean-revert to any long term level after a credit crisis.

Despite these simplistic and questionable assumptions about the credit spread dynamics, we actually believe that the outcomes of the model, displayed in Table 3[8], are insightful. The discrepancy between the equity tranche deltas in the Markov chain and in the Gaussian copula model can be easily explained by a dynamic correlation effect. The arrival of defaults is associated with contagion effects, thus an increase of dependence, which is actually taken in account in the Markov chain model but not in the Gaussian copula approach. This tends to lower the equity tranche deltas.

	[0-3%]	[3-6%]	[6-9%]	[9-12%]	[12-22%]
market deltas	27	4.5	1.25	0.6	0.25
model deltas	21.5	4.63	1.63	0.9	0.6

Table 3. Market delta spreads and model deltas (a default event) at inception.

Figure 5 shows the base correlation curves at a 14 weeks horizon, when the number of defaults is equal to zero, one or two. We can see that the arrival of the first defaults is associated with parallel shifts in the base correlation curves.

[7] After a default, the loss intensity will be lower if a high yield name is involved than if a low credit spread name defaults. The Markov property of the aggregate loss process might be preserved provided that we deal with large portfolios, since random thinning (see Giesecke and Goldberg (2005)) could be seen as a sampling scheme with replacement. For standard tranches, this might be an acceptable approximation but further theoretical and empirical researches should be considered before one could validate the approach.

[8] See Laurent, Cousin and Fermanian (2007).

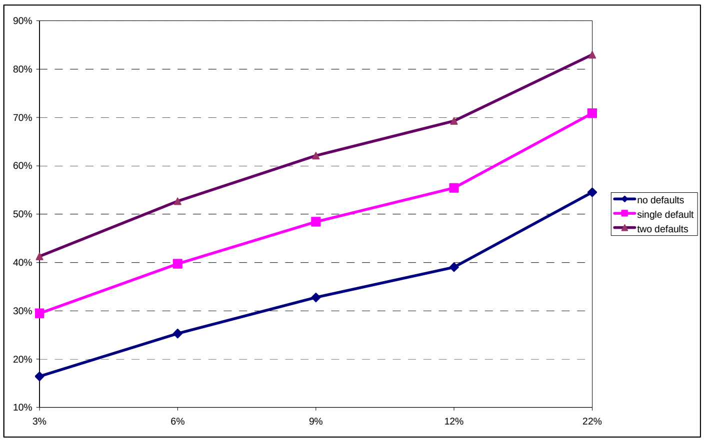

Figure 5. Dynamics of the base correlation curve with respect to the number of defaults. Detachment points on the x–axis. Base correlations on the y–axis.

The deltas associated with the Markov chain model correspond to the "sticky implied tree model" deltas of Derman (1999). These are suitable in a regime of fear, corresponding to systemic credit shifts. The 2007 credit crisis shows empirical evidence of such co-movements of correlation and credit spreads reflecting the likelihood of future defaults. Figure 6 shows the dynamics of the five year iTraxx credit spread and of the implied correlation of the equity tranche. Over this period the linear correlation between the two series was equal to 91%. This clearly favours the contagion model and suggests a flaw in current market practice.

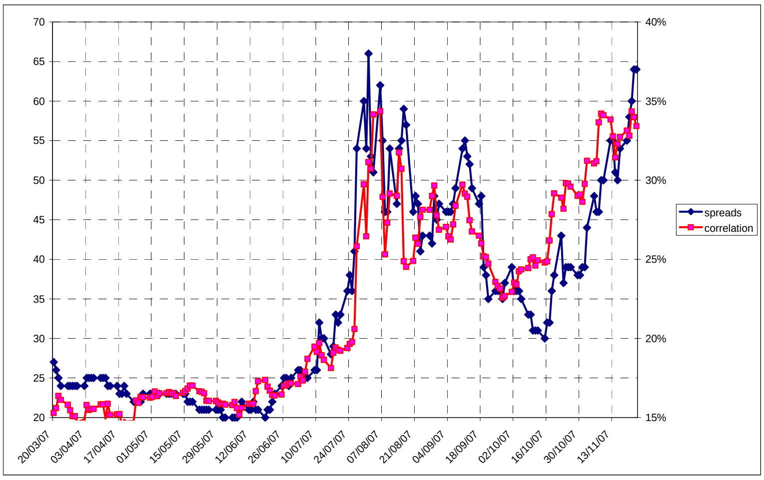

Figure 6. Credit spreads on the five years iTraxx index (Series 7) in bps on the left axis. Implied correlation on the equity tranche on the right axis.

Dealing with contagion effects is not specific to the Markov chain model. Schönbucher and Schubert (2001) have shown that copula models exhibit some contagion effects and relate jumps of credit spreads at default times to the partial derivatives of the copula. This is also the framework used by Bielecki, Jeanblanc and Rutkowski (2007) to address the hedging issue. They firstly consider the hedging of a first to default swap and subsequently use a recursive approach for the hedging of more general basket credit derivatives. On theoretical grounds, there are no significant differences with the approach of Frey and Backhaus (2007a) or Laurent, Cousin and Fermanian (2007). In any case, the arrival of defaults is the only driver of the dynamics of the loss and credit spread processes. Between two default dates, credit spreads of the survival names are deterministic. However, in copula models, the loss process is usually not Markovian and the dynamics of the credit spreads depends on the complete history of default dates and not only on the number of defaults as in the Markov chain approach. This should lead to extra-numerical complexity, but this is still to be assessed.

Eventually, the deltas with respect to credit default swaps computed in the Markov chain model are associated with the arrival of defaults which are rare events[9]. Market participants thus favour hedging exposures of trading books to daily changes in credit spreads. Though this is not anymore related to a replicating strategy, it is possible to use the Markov chain model as a pricing tool and compute sensitivities with respect to market inputs such as credit default swap index spreads as discussed in Section I or liquid tranche quotes as discussed in Section II. Fortunately enough, as can be seen from Table 4, the resulting deltas are quite close to the theoretical default deltas shown in Table 3.

Tranches	[0-3%]	[3-6%]	[6-9%]	[9-12%]	[12-22%]
Delta Bump Intensities	21.64	5.01	1.55	0.80	0.51

Table 4. Deltas obtained by bumping the loss intensities.

IV) Paths for the future

We describe here some ideas and techniques that are still in progress but may prove to be fruitful for practitioners in a near future.

IV.1) Using credit default swaps of different maturities

A clear weakness of the approach described in Section III is that credit spreads are deterministic between two default dates. It is therefore desirable to consider models that account both for default and credit spread risks. Since there is an extra-source of risk, one either needs to cope with market incompleteness or introduce more hedging instruments. A simple idea is to use single name credit default swaps of two maturities in order to be hedged both against the arrival of defaults and against changes in credit spreads between the arrival of defaults. Bielecki, Jeanblanc and Rutkowski (2007) provide the mathematical foundation for the replication of CDO tranches in such a framework. Becherer and Schweizer (2005) tackle the problem from a slightly different perspective. However, the practical implementation of such ideas remains to be done. It is yet unclear what would be the balance between long and short term default swaps in the resulting hedging strategy. Since both defaults and credit spreads

[9] Let us stress that the exposure at default as computed in the Markov chain model is not equal to the usual "value on default" or iOmega. In the Markov chain, the arrival of default is associated with a shift in credit spreads and in base correlations due to contagion effects, while the value on default is usually computed under the assumption of constant spreads and correlations.

drive the expected loss on the underlying portfolio, it may well be that these two risks are rather "parallel". If this would be true, one could at the same time hedge credit spread and default risks, thus the use of credit default swaps of different maturities would only slightly increase the hedging efficiency. Clearly, it is too early to give a definitive answer and further research should be conducted to assess the practical improvements due to the use of a more realistic model.

IV.2) Local hedging approaches

Due to the complexity of the building of replicating strategies when dealing both with credit spread and default risks, one could think of using local risk minimization techniques as in Frey and Backhaus (2007b). Elouerkhaoui (2006) uses similar techniques though the pricing framework is quite different. In the latter approach, the dependence between defaults is due to the occurrence of simultaneous defaults. We also refer to Bielecki, Jeanblanc and Rutkowski (2004a, 2004b, 2006), Petrelli, Siu, Zhang and Kapoor (2006) for further discussion on this topic.

IV.3) Asymptotic hedging

This approach also aims at solving the default and credit spread hedging issue. It is noticeable that market participants focus on the day to day hedging of credit spread risks and tend to neglect default risks. Let us however remark that the direct effect of a default event on the value of an iTraxx or CDX portfolio is quite small due to the number of names involved compared for example with daily changes in stock indices. In other words, there is a diversification effect of default risk at the portfolio level. For an infinitely granular portfolio, losses would be only driven by changes in credit spreads. Such a diversification of default risk will actually occur for large portfolios provided that defaults do not lead to shifts in credit spreads of the survival names. This corresponds to the assumption of conditional independence of default events upon the default intensities, or equivalently to the multivariate Cox process framework. This is illustrated for instance by the affine modelling of Eckner (2007) discussed above. In this framework, defaults are non informative or stated otherwise there are no contagion effects contrary to the Markov chain model. Laurent (2006) has investigated such a framework and shown that one could consider the dynamic hedging of credit spread risks only, while keeping the hedging error under control. Default risk is managed on its own at the underlying portfolio level. Thus the approach mixes techniques from modern mathematical finance such as dynamic hedging and older ideas related to the management of insurance risks. Greenberg, O'Kane and Schloegl (2004) compute credit spread sensitivities using a large portfolio approximation in a copula framework. Since the copula setting is not associated with multivariate Cox processes, the resulting computations should be viewed as a quick and efficient way to compute sensitivities as discussed in Section I and are not related to replication ideas.

Conclusion

Up to now, mainstream research in the CDO field was focused on pricing models. While still a lot of work remains to be done in that direction, we think that the market calls for improvements with respect to risk management issues. This is a hard but challenging task both for academics and quantitative analysts. Preliminary results hopefully show that hedging approaches based on different models might eventually lead to oddly similar empirical hedging strategies. While this needs to be confirmed, it suggests that there is actually room to construct efficient risk mitigating strategies, which is eventually the reasonable target.

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