Hedging default risk of CDOs in Markovian contagion models

Areski Cousin
Université Claude Bernard Lyon 1, ISFA

Séminaire Lyon-Lausanne
26 Novembre 2007

Presentation related to paper:
Hedging default risk of CDOs in Markovian contagion models (2007)
Joint work with Jean-Paul Laurent and Jean-David Fermanian
Available on www.defaultrisk.com
In interest rate or equity markets, pricing is related to the cost of the hedge
  - ex: Blask-Scholes pricing model of equity options

In credit markets, pricing is disconnect from hedging
  - ex: The industrial CDO pricing model, use of local hedging strategies

Need to relate pricing and hedging

In defaultrisk.com
  - More than 1000 papers
  - About 10 papers deal with hedging issues
• Purpose of the presentation
  ➢ Not trying to embrace all risk management issues
  ➢ Focus on very specific aspects of default and credit spread risk
  ➢ Obtain replication strategies for CDO tranches

• Overlook of the presentation
  ➢ Tree approach to hedging defaults
  ➢ Results and comments
Tree approach to hedging defaults

• We will start with two names only
• Firstly in a static framework
  – Hedging a First to Default Swap
  – Discuss historical and risk-neutral probabilities
• Further extending the model to a dynamic framework
  – Computation of prices and hedging strategies along the tree
  – Pricing and hedging of zero coupon CDO tranches
• Multiname case: homogeneous Markovian model
  – Computation of risk-neutral tree for the loss
  – Computation of dynamic deltas
• Technical details can be found in the paper:
  – “hedging default risks of CDOs in Markovian contagion models”
**Tree approach to hedging defaults**

- Some notations:
  - $\tau_1, \tau_2$ default times of counterparties 1 and 2,
  - $\mathcal{H}_t$ available information at time $t$,
  - $P$ historical probability,
  - $\alpha_1^p, \alpha_2^p$ : (historical) default intensities:
    - $P\left[\tau_i \in [t, t+dt] \mid \mathcal{H}_t\right] = \alpha_i^p dt$, $i = 1, 2$

- Assumption of « local » independence between default events
  - Probability of 1 and 2 defaulting altogether:
    - $P\left[\tau_1 \in [t, t+dt], \tau_2 \in [t, t+dt] \mid \mathcal{H}_t\right] = \alpha_1^p dt \times \alpha_2^p dt \text{ in } (dt)^2$
  - Local independence: simultaneous joint defaults can be neglected
Tree approach to hedging defaults

- Building up a tree:
  - Four possible states: \((D,D), (D,ND), (ND,D), (ND,ND)\)
  - Under no simultaneous defaults assumption \(p_{(D,D)}=0\)
  - Only three possible states: \((D,ND), (ND,D), (ND,ND)\)
  - Identifying (historical) tree probabilities:

\[
\begin{align*}
\alpha_1^p \, dt & \quad (D,ND) \\
\alpha_2^p \, dt & \quad (ND,D) \\
1 - (\alpha_1^p + \alpha_2^p) \, dt & \quad (ND,ND)
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
 p_{(D,D)} = 0 \Rightarrow p_{(D,ND)} = p_{(D,D)} + p_{(D,ND)} = p_{(D,.)} = \alpha_1^p \, dt \\
p_{(D,D)} = 0 \Rightarrow p_{(ND,D)} = p_{(D,D)} + p_{(ND,D)} = p_{(.,D)} = \alpha_2^p \, dt \\
p_{(ND,ND)} = 1 - p_{(D,.)} - p_{(.,D)}
\end{cases}
\end{align*}
\]
Tree approach to hedging defaults

- Stylized cash flows of short term digital CDS on counterparty 1:
  - CDS 1 premium $\alpha_1^Q dt$

    \[
    \begin{array}{c}
    \alpha_1^P dt \\
    \alpha_2^P dt \\
    0 \\
    \end{array}
    \begin{array}{c}
    1 - \alpha_1^Q dt \\
    -\alpha_1^Q dt \\
    -\alpha_1^Q dt \\
    \end{array}
    \begin{array}{c}
    (D, ND) \\
    (ND, D) \\
    (ND, ND) \\
    \end{array}
    \]

  - Stylized cash flows of short term digital CDS on counterparty 2:

    \[
    \begin{array}{c}
    \alpha_1^P dt \\
    \alpha_2^P dt \\
    0 \\
    \end{array}
    \begin{array}{c}
    -\alpha_2^Q dt \\
    1 - \alpha_2^Q dt \\
    1 - (\alpha_1^P + \alpha_2^P) dt \\
    \end{array}
    \begin{array}{c}
    (D, ND) \\
    (ND, D) \\
    (ND, ND) \\
    \end{array}
    \]
**Tree approach to hedging defaults**

- Cash flows of short term digital first to default swap with premium $\alpha_F^O dt$:
  
  $\alpha_1^P dt$ \\
  $\alpha_2^P dt$ \\
  $1 - \alpha_F^O dt$ (D, ND) \\
  $1 - \left(\alpha_1^P + \alpha_2^P\right) dt$ \\
  $1 - \alpha_F^O dt$ (ND, D) \\
  $1 - \left(\alpha_1^P + \alpha_2^P\right) dt$ \\
  $-\alpha_F^O dt$ (ND, ND)

- Cash flows of holding CDS 1 + CDS 2:
  
  $\alpha_1^P dt$ \\
  $\alpha_2^P dt$ \\
  $1 - \left(\alpha_1^O + \alpha_2^O\right) dt$ (D, ND) \\
  $1 - \left(\alpha_1^O + \alpha_2^O\right) dt$ (ND, D) \\
  $1 - \left(\alpha_1^O + \alpha_2^O\right) dt$ (ND, ND)

- Perfect hedge of first to default swap by holding 1 CDS 1 + 1 CDS 2
  
  Delta with respect to CDS 1 = 1, delta with respect to CDS 2 = 1
Absence of arbitrage opportunities imply:
\[ \alpha_F^Q = \alpha_1^Q + \alpha_2^Q \]

Arbitrage free first to default swap premium
\[ \text{Does not depend on historical probabilities } \alpha_1^p, \alpha_2^p \]

Three possible states: \((D, ND), (ND, D), (ND, ND)\)

Three tradable assets: CDS1, CDS2, risk-free asset

For simplicity, let us assume \( r = 0 \)
Three state contingent claims

- Example: claim contingent on state \((D, ND)\)
- Can be replicated by holding
  - \(\alpha_1^0 dt\) risk-free asset + 1 CDS 1

Replication price = \(\alpha_1^0 dt\)
Tree approach to hedging defaults

- Similarly, the replication prices of the \((ND, D)\) and \((ND, ND)\) claims

\[
\begin{align*}
\alpha_2^O \, dt & \quad 0 \quad (D, \text{ND}) \\
\alpha_2^P \, dt & \quad 1 \quad (ND, D) \\
\alpha_2^o \, dt & \quad 1 - (\alpha_1^o + \alpha_2^o) \, dt \quad (ND, ND)
\end{align*}
\]

- Replication price of:

\[
\begin{align*}
\alpha_1^P \, dt & \quad a \quad (D, \text{ND}) \\
\alpha_2^P \, dt & \quad b \quad (ND, D) \\
\alpha_2^o \, dt & \quad c \quad (ND, ND)
\end{align*}
\]

- Replication price = \(a \times \alpha_1^o \, dt + b \times \alpha_2^o \, dt + c \times (1 - (\alpha_1^o + \alpha_2^o) \, dt)\)
Tree approach to hedging defaults

• Replication price obtained by computing the expected payoff
  – Along a risk-neutral tree

\[
\alpha_1^Q dt \times a + \alpha_2^Q dt \times b + \left( 1 - (\alpha_1^Q + \alpha_2^Q) dt \right) c
\]

\[
1 - (\alpha_1^Q + \alpha_2^Q) dt
\]

• Risk-neutral probabilities
  – Used for computing replication prices
  – Uniquely determined from short term CDS premiums
  – No need of historical default probabilities
• Computation of deltas
  – Delta with respect to risk-free asset: $\delta_0$
  – Delta with respect to CDS 1: $\delta_1$
  – Delta with respect to CDS 2: $\delta_2$

\[
\begin{align*}
  a &= \delta_0 + \delta_1 \times (1 - \alpha_1^Q dt) + \delta_2 \times (-\alpha_2^Q dt) \\
  b &= \delta_0 + \delta_1 \times (-\alpha_1^Q dt) + \delta_2 \times (1 - \alpha_2^Q dt) \\
  c &= \delta_0 + \delta_1 \times (-\alpha_1^Q dt) + \delta_2 \times (-\alpha_2^Q dt)
\end{align*}
\]

– As for the replication price, deltas only depend upon CDS premiums
Dynamic case:

- $\lambda_2^Q dt$  CDS 2 premium after default of name 1
- $\lambda_1^Q dt$  CDS 1 premium after default of name 2
- $\pi_1^Q dt$  CDS 1 premium if no name defaults at period 1
- $\pi_2^Q dt$  CDS 2 premium if no name defaults at period 1

Change in CDS premiums due to contagion effects

- Usually, $\pi_1^Q < \alpha_1^Q < \lambda_1^Q$ and $\pi_2^Q < \alpha_2^Q < \lambda_2^Q
Computation of prices and hedging strategies by backward induction

- use of the dynamic risk-neutral tree
- Start from period 2, compute price at period 1 for the three possible nodes
- + hedge ratios in short term CDS 1,2 at period 1
- Compute price and hedge ratio in short term CDS 1,2 at time 0

Example to be detailed:

- computation of CDS 1 premium, maturity = 2
- \( p_1 dt \) will denote the periodic premium
- Cash-flow along the nodes of the tree
Tree approach to hedging defaults

- Computations CDS on name 1, maturity = 2

\[ 0 = \left( 1 - p_1 \right) \alpha_1^0 + \left( -p_1 + \left( 1 - p_1 \right) \lambda_1^0 - p_1 \left( 1 - \lambda_1^0 \right) \right) \alpha_2^0 \]
\[ + \left( -p_1 + \left( 1 - p_1 \right) \pi_1^0 - p_1 \pi_2^0 - p_1 \left( 1 - \pi_1^0 - \pi_2^0 \right) \right) \left( 1 - \alpha_1^0 - \alpha_2^0 \right) \]

- Premium of CDS on name 1, maturity = 2, time = 0, \( dt = 1 \), \( p_1 \) solves for:
Example: stylized zero coupon CDO tranchelets

- Zero-recovery, maturity 2
- Aggregate loss at time 2 can be equal to 0, 1, 2
  - Equity type tranche contingent on no defaults
  - Mezzanine type tranche: one default
  - Senior type tranche: two defaults

Tree approach to hedging defaults
Tree approach to hedging defaults

- mezzanine tranche
  - Time pattern of default payments
    - Possibility of taking into account discounting effects
    - The timing of premium payments
    - Computation of dynamic deltas with respect to short or actual CDS on names 1,2

\[
\alpha_1^o \, dt + \alpha_2^o \, dt + \left(1 - (\alpha_1^o + \alpha_2^o) dt\right) \left(\pi_1^o + \pi_2^o\right) dt
\]
• In theory, one could also derive dynamic hedging strategies for index CDO tranches
  – Numerical issues: large dimensional, non recombining trees
  – Homogeneous Markovian assumption is very convenient

➢ CDS premiums at a given time $t$ only depend upon the current number of defaults $N(t)$
  – CDS premium at time 0 (no defaults) $\alpha^0_1 \ dt = \alpha^0_2 \ dt = \alpha^0 (t = 0, N(0) = 0)$
  – CDS premium at time 1 (one default) $\lambda^0_1 \ dt = \lambda^0_2 \ dt = \alpha^0 (t = 1, N(t) = 1)$
  – CDS premium at time 1 (no defaults) $\pi^0_1 \ dt = \pi^0_2 \ dt = \alpha^0 (t = 1, N(t) = 0)$
Tree approach to hedging defaults

- Homogeneous Markovian tree

\[ \begin{align*}
\alpha_0^Q (0,0) & \quad (D,ND) \\
\alpha_0^Q (0,0) & \quad (ND,D) \\
1 - 2\alpha_1^Q (0,0) & \quad (ND,ND) \\
\end{align*} \]

- If we have \( N(1) = 1 \), one default at \( t=1 \)
- The probability to have \( N(2) = 1 \), one default at \( t=2 \)…
- Is \( 1 - \alpha_1^Q (1,1) \) and does not depend on the defaulted name at \( t=1 \)
- \( N(t) \) is a Markov process
- Dynamics of the number of defaults can be expressed through a binomial tree
Tree approach to hedging defaults

From name per name to number of defaults tree

\[
\begin{align*}
\alpha^Q(0,0) & \quad (D, ND) \\
1 - 2\alpha^Q(0,0) & \quad (ND, D) \\
\alpha^Q(1,1) & \quad (D, D) \\
1 - \alpha^Q(1,1) & \quad (D, ND)
\end{align*}
\]

\[
\begin{align*}
\alpha^Q(0,0) & \quad (ND, D) \\
\alpha^Q(1,0) & \quad (D, ND) \\
1 - 2\alpha^Q(1,0) & \quad (ND, ND)
\end{align*}
\]

\[
\begin{align*}
\alpha^Q(0,0) & \quad (ND, D) \\
\alpha^Q(1,1) & \quad (D, D) \\
1 - \alpha^Q(1,1) & \quad (D, ND)
\end{align*}
\]

\[
\begin{align*}
\alpha^Q(0,0) & \quad (ND, ND) \\
\alpha^Q(1,0) & \quad (D, ND) \\
1 - 2\alpha^Q(1,0) & \quad (ND, ND)
\end{align*}
\]

Number of defaults tree

- \( N(0) = 0 \) leads to \( 2\alpha^Q(0,0) \) (with \( N(1) = 1 \) and \( N(2) = 1 \))
- \( N(1) = 0 \) leads to \( 1 - 2\alpha^Q(0,0) \) (with \( N(2) = 0 \))
- \( N(0) = 0 \) leads to \( 2\alpha^Q(1,0) \) (with \( N(1) = 0 \) and \( N(2) = 0 \))
- \( N(1) = 1 \) leads to \( 1 - \alpha^Q(1,1) \) (with \( N(2) = 2 \))
- \( N(1) = 1 \) leads to \( 2\alpha^Q(1,0) \) (with \( N(2) = 1 \))
**Tree approach to hedging defaults**

- Easy extension to $n$ names
  - Predefault name intensity at time $t$ for $N(t)$ defaults: $\alpha^Q_i (t, N(t))$
  - Number of defaults intensity: sum of surviving name intensities:
    \[ \lambda(t, N(t)) = (n - N(t)) \alpha^Q_i (t, N(t)) \]

\[ \begin{align*}
N(0) &= 0 \\
N(1) &= 1 \\
N(2) &= 2 \\
N(3) &= 3
\end{align*} \]

\[ \begin{align*}
1 - n \alpha^Q_i (0,0) &\quad N(1) = 1 \\
1 - (n - 1) \alpha^Q_i (1,1) &\quad N(2) = 2 \\
1 - (n - 1) \alpha^Q_i (2,2) &\quad N(3) = 3
\end{align*} \]

- $\alpha^Q_i (0,0), \alpha^Q_i (1,0), \alpha^Q_i (1,1), \alpha^Q_i (2,0), \alpha^Q_i (2,1)$, … can be easily calibrated
- on marginal distributions of $N(t)$ by forward induction.
Tree approach to hedging defaults

• Previous recombining binomial risk-neutral tree provides a framework for the valuation of payoffs depending upon the number of defaults

CDS Index: homogeneous portfolio of CDS
  - Cash-flows contingent to the aggregate loss process \( L_t = (1 - \delta) \frac{N_t}{n} \)
  - where \( \delta \) is the recovery rate

CDO tranche \([a\%, b\%]\)
  - Cash-flows contingent to \( L_t^{(a,b)} \)
  - Call spread option on the aggregate loss
What about the credit deltas?

- In a homogeneous framework, deltas with respect to CDS are all the same
- Perfect dynamic replication of a CDO tranche with a credit default swap index and the default-free asset
- Credit delta with respect to the credit default swap index
- \( = \frac{\text{change in PV of the tranche}}{\text{change in PV of the CDS index}} \)
Input: number of defaults distribution at 5Y generated from a Gaussian copula

- Number of names: 125
- Correlation parameter: 30%
- Default-free rate: 3%
- CDS spreads: 20bps per annum
  - 5Y default probability: 1.65%
- Recovery rate: 40%
**Results and comments**

- **Calibration of loss intensities**
  - For simplicity, assumption of time homogeneous intensities
  - Figure below represents loss intensities, with respect to the number of defaults
  - Increase in intensities: contagion effects

![Graph showing lambda Gauss with k (Number of defaults) on the x-axis and lambda (λk) on the y-axis.](image)
Results and comments

- Dynamics of the 5Y CDS index spread
  - In bp pa

<table>
<thead>
<tr>
<th>Nb Defaults</th>
<th>Weeks</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>14</td>
<td>28</td>
<td>42</td>
<td>56</td>
<td>70</td>
<td>84</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
<td>19</td>
<td>19</td>
<td>18</td>
<td>18</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>31</td>
<td>30</td>
<td>29</td>
<td>28</td>
<td>27</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>46</td>
<td>44</td>
<td>43</td>
<td>41</td>
<td>40</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>63</td>
<td>61</td>
<td>58</td>
<td>56</td>
<td>54</td>
<td>52</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>83</td>
<td>79</td>
<td>76</td>
<td>73</td>
<td>70</td>
<td>67</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>104</td>
<td>99</td>
<td>95</td>
<td>91</td>
<td>87</td>
<td>83</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>127</td>
<td>121</td>
<td>116</td>
<td>111</td>
<td>106</td>
<td>101</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>151</td>
<td>144</td>
<td>138</td>
<td>132</td>
<td>126</td>
<td>120</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>176</td>
<td>169</td>
<td>161</td>
<td>154</td>
<td>146</td>
<td>140</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>203</td>
<td>194</td>
<td>185</td>
<td>176</td>
<td>168</td>
<td>160</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>230</td>
<td>219</td>
<td>209</td>
<td>200</td>
<td>190</td>
<td>181</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>257</td>
<td>246</td>
<td>235</td>
<td>224</td>
<td>213</td>
<td>203</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>284</td>
<td>272</td>
<td>260</td>
<td>248</td>
<td>237</td>
<td>225</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>310</td>
<td>298</td>
<td>286</td>
<td>273</td>
<td>260</td>
<td>248</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>336</td>
<td>324</td>
<td>311</td>
<td>298</td>
<td>284</td>
<td>271</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>348</td>
<td>336</td>
<td>323</td>
<td>308</td>
<td>294</td>
</tr>
</tbody>
</table>
Dynamics of credit deltas:

- \([0, 3\%]\) equity tranche
- With respect to the 5Y CDS index
- For selected time steps

- Hedging strategy leads to a perfect replication of equity tranche payoff
- When the number of defaults is > 6, the tranche is exhausted
Results and comments

<table>
<thead>
<tr>
<th>Nb Defaults</th>
<th>Outstanding Nominal</th>
<th>Weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>0</td>
<td>3.00%</td>
<td>0.814</td>
</tr>
<tr>
<td>1</td>
<td>2.52%</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2.04%</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.56%</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1.08%</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.60%</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.12%</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.00%</td>
<td>0</td>
</tr>
</tbody>
</table>

- Deltas are actually between 0 and 1
- Gradually decrease with the number of defaults
  - Concave payoff, negative gammas
- Credit deltas increase with time
  - Consistent with a decrease in time value
  - At maturity date, when number of defaults < 6, delta=1
Results and comments

- Dynamics of credit deltas
  - **Junior mezzanine tranche [3.6%]**
  - Deltas lie in between 0 and 1
  - When the number of defaults is above 12, the tranche is exhausted

<table>
<thead>
<tr>
<th>Nb Defaults</th>
<th>OutStanding Nominal</th>
<th>0</th>
<th>14</th>
<th>28</th>
<th>42</th>
<th>56</th>
<th>70</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.00%</td>
<td>0.162</td>
<td>0.139</td>
<td>0.117</td>
<td>0.096</td>
<td>0.077</td>
<td>0.059</td>
<td>0.045</td>
</tr>
<tr>
<td>1</td>
<td>3.00%</td>
<td>0</td>
<td>0.327</td>
<td>0.298</td>
<td>0.266</td>
<td>0.232</td>
<td>0.197</td>
<td>0.162</td>
</tr>
<tr>
<td>2</td>
<td>3.00%</td>
<td>0</td>
<td>0.497</td>
<td>0.489</td>
<td>0.473</td>
<td>0.448</td>
<td>0.415</td>
<td>0.376</td>
</tr>
<tr>
<td>3</td>
<td>3.00%</td>
<td>0</td>
<td>0.521</td>
<td>0.552</td>
<td>0.576</td>
<td>0.591</td>
<td>0.595</td>
<td>0.586</td>
</tr>
<tr>
<td>4</td>
<td>3.00%</td>
<td>0</td>
<td>0.400</td>
<td>0.454</td>
<td>0.508</td>
<td>0.562</td>
<td>0.611</td>
<td>0.652</td>
</tr>
<tr>
<td>5</td>
<td>3.00%</td>
<td>0</td>
<td>0.239</td>
<td>0.288</td>
<td>0.343</td>
<td>0.405</td>
<td>0.473</td>
<td>0.544</td>
</tr>
<tr>
<td>6</td>
<td>3.00%</td>
<td>0</td>
<td>0.123</td>
<td>0.153</td>
<td>0.190</td>
<td>0.236</td>
<td>0.291</td>
<td>0.358</td>
</tr>
<tr>
<td>7</td>
<td>2.64%</td>
<td>0</td>
<td>0.059</td>
<td>0.073</td>
<td>0.090</td>
<td>0.115</td>
<td>0.147</td>
<td>0.189</td>
</tr>
<tr>
<td>8</td>
<td>2.16%</td>
<td>0</td>
<td>0.031</td>
<td>0.036</td>
<td>0.043</td>
<td>0.052</td>
<td>0.066</td>
<td>0.086</td>
</tr>
<tr>
<td>9</td>
<td>1.68%</td>
<td>0</td>
<td>0.019</td>
<td>0.020</td>
<td>0.023</td>
<td>0.026</td>
<td>0.030</td>
<td>0.037</td>
</tr>
<tr>
<td>10</td>
<td>1.20%</td>
<td>0</td>
<td>0.012</td>
<td>0.012</td>
<td>0.013</td>
<td>0.014</td>
<td>0.016</td>
<td>0.018</td>
</tr>
<tr>
<td>11</td>
<td>0.72%</td>
<td>0</td>
<td>0.007</td>
<td>0.007</td>
<td>0.007</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td>12</td>
<td>0.24%</td>
<td>0</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>13</td>
<td>0.00%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Dynamics of credit deltas (junior mezzanine tranche)
- Gradually increase and then decrease with the number of defaults
- Call spread payoff (convex, then concave)
- Initial delta = 16% (out of the money option)

<table>
<thead>
<tr>
<th>Nb Defaults</th>
<th>OutStanding Nominal</th>
<th>0</th>
<th>14</th>
<th>28</th>
<th>42</th>
<th>56</th>
<th>70</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.00%</td>
<td>0.162</td>
<td>0.139</td>
<td>0.117</td>
<td>0.096</td>
<td>0.077</td>
<td>0.059</td>
<td>0.045</td>
</tr>
<tr>
<td>1</td>
<td>3.00%</td>
<td>0</td>
<td>0.327</td>
<td>0.298</td>
<td>0.266</td>
<td>0.232</td>
<td>0.197</td>
<td>0.162</td>
</tr>
<tr>
<td>2</td>
<td>3.00%</td>
<td>0</td>
<td>0.497</td>
<td>0.489</td>
<td>0.473</td>
<td>0.448</td>
<td>0.415</td>
<td>0.376</td>
</tr>
<tr>
<td>3</td>
<td>3.00%</td>
<td>0</td>
<td>0.521</td>
<td>0.552</td>
<td>0.576</td>
<td>0.591</td>
<td>0.595</td>
<td>0.586</td>
</tr>
<tr>
<td>4</td>
<td>3.00%</td>
<td>0</td>
<td>0.400</td>
<td>0.454</td>
<td>0.508</td>
<td>0.562</td>
<td>0.611</td>
<td>0.652</td>
</tr>
<tr>
<td>5</td>
<td>3.00%</td>
<td>0</td>
<td>0.239</td>
<td>0.288</td>
<td>0.343</td>
<td>0.405</td>
<td>0.473</td>
<td>0.544</td>
</tr>
<tr>
<td>6</td>
<td>3.00%</td>
<td>0</td>
<td>0.123</td>
<td>0.153</td>
<td>0.190</td>
<td>0.236</td>
<td>0.291</td>
<td>0.358</td>
</tr>
<tr>
<td>7</td>
<td>2.64%</td>
<td>0</td>
<td>0.059</td>
<td>0.073</td>
<td>0.090</td>
<td>0.115</td>
<td>0.147</td>
<td>0.189</td>
</tr>
<tr>
<td>8</td>
<td>2.16%</td>
<td>0</td>
<td>0.031</td>
<td>0.036</td>
<td>0.043</td>
<td>0.052</td>
<td>0.066</td>
<td>0.086</td>
</tr>
<tr>
<td>9</td>
<td>1.68%</td>
<td>0</td>
<td>0.019</td>
<td>0.020</td>
<td>0.023</td>
<td>0.026</td>
<td>0.030</td>
<td>0.037</td>
</tr>
<tr>
<td>10</td>
<td>1.20%</td>
<td>0</td>
<td>0.012</td>
<td>0.012</td>
<td>0.013</td>
<td>0.014</td>
<td>0.016</td>
<td>0.018</td>
</tr>
<tr>
<td>11</td>
<td>0.72%</td>
<td>0</td>
<td>0.007</td>
<td>0.007</td>
<td>0.007</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td>12</td>
<td>0.24%</td>
<td>0</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>13</td>
<td>0.00%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Dynamics of credit deltas ([6.9%] tranche)
- Initial credit deltas are smaller (further out of the money call spread)

<table>
<thead>
<tr>
<th>Nb Defaults</th>
<th>OutStanding Nominal</th>
<th>0</th>
<th>14</th>
<th>28</th>
<th>42</th>
<th>56</th>
<th>70</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.00%</td>
<td>0.017</td>
<td>0.012</td>
<td>0.008</td>
<td>0.005</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>1</td>
<td>3.00%</td>
<td>0</td>
<td>0.048</td>
<td>0.036</td>
<td>0.025</td>
<td>0.017</td>
<td>0.011</td>
<td>0.006</td>
</tr>
<tr>
<td>2</td>
<td>3.00%</td>
<td>0</td>
<td>0.133</td>
<td>0.107</td>
<td>0.083</td>
<td>0.061</td>
<td>0.043</td>
<td>0.029</td>
</tr>
<tr>
<td>3</td>
<td>3.00%</td>
<td>0</td>
<td>0.259</td>
<td>0.227</td>
<td>0.193</td>
<td>0.157</td>
<td>0.122</td>
<td>0.090</td>
</tr>
<tr>
<td>4</td>
<td>3.00%</td>
<td>0</td>
<td>0.371</td>
<td>0.356</td>
<td>0.330</td>
<td>0.295</td>
<td>0.253</td>
<td>0.206</td>
</tr>
<tr>
<td>5</td>
<td>3.00%</td>
<td>0</td>
<td>0.405</td>
<td>0.423</td>
<td>0.428</td>
<td>0.420</td>
<td>0.396</td>
<td>0.358</td>
</tr>
<tr>
<td>6</td>
<td>3.00%</td>
<td>0</td>
<td>0.346</td>
<td>0.392</td>
<td>0.433</td>
<td>0.465</td>
<td>0.482</td>
<td>0.481</td>
</tr>
<tr>
<td>7</td>
<td>3.00%</td>
<td>0</td>
<td>0.239</td>
<td>0.292</td>
<td>0.350</td>
<td>0.409</td>
<td>0.465</td>
<td>0.510</td>
</tr>
<tr>
<td>8</td>
<td>3.00%</td>
<td>0</td>
<td>0.139</td>
<td>0.181</td>
<td>0.232</td>
<td>0.293</td>
<td>0.363</td>
<td>0.436</td>
</tr>
<tr>
<td>9</td>
<td>3.00%</td>
<td>0</td>
<td>0.074</td>
<td>0.098</td>
<td>0.132</td>
<td>0.177</td>
<td>0.235</td>
<td>0.307</td>
</tr>
<tr>
<td>10</td>
<td>3.00%</td>
<td>0</td>
<td>0.042</td>
<td>0.053</td>
<td>0.070</td>
<td>0.095</td>
<td>0.132</td>
<td>0.183</td>
</tr>
<tr>
<td>11</td>
<td>3.00%</td>
<td>0</td>
<td>0.029</td>
<td>0.033</td>
<td>0.040</td>
<td>0.051</td>
<td>0.070</td>
<td>0.098</td>
</tr>
<tr>
<td>12</td>
<td>3.00%</td>
<td>0</td>
<td>0.025</td>
<td>0.026</td>
<td>0.028</td>
<td>0.033</td>
<td>0.040</td>
<td>0.053</td>
</tr>
<tr>
<td>13</td>
<td>2.76%</td>
<td>0</td>
<td>0.022</td>
<td>0.022</td>
<td>0.022</td>
<td>0.024</td>
<td>0.026</td>
<td>0.031</td>
</tr>
<tr>
<td>14</td>
<td>2.28%</td>
<td>0</td>
<td>0.020</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
<td>0.019</td>
<td>0.020</td>
</tr>
<tr>
<td>15</td>
<td>1.80%</td>
<td>0</td>
<td>0</td>
<td>0.015</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td>16</td>
<td>1.32%</td>
<td>0</td>
<td>0</td>
<td>0.013</td>
<td>0.011</td>
<td>0.010</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>17</td>
<td>0.84%</td>
<td>0</td>
<td>0</td>
<td>0.009</td>
<td>0.008</td>
<td>0.007</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>18</td>
<td>0.36%</td>
<td>0</td>
<td>0</td>
<td>0.005</td>
<td>0.004</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>19</td>
<td>0.00%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
What do we learn from this hedging approach?

- Thanks to stringent assumptions:
  - credit spreads driven by defaults
  - homogeneity
  - Markov property
- It is possible to compute a dynamic hedging strategy
  - Based on the CDS index
- That fully replicates the CDO tranche payoffs
  - Very simple implementation
  - Credit deltas are easy to understand
- Credit spread dynamics needs to be improved