Delta-Hedging Correlation Risk?

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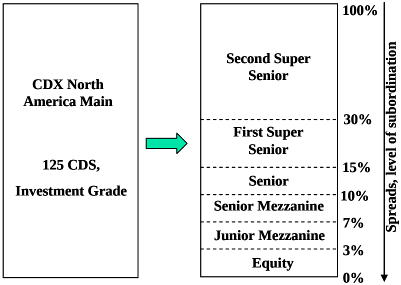
Introduction

Rama Cont, Areski Cousin, Stéphane Crépey and Yu Hang Kan (2010)
Delta-Hedging Correlation Risk?

Introduction

- Performance analysis of alternative hedging strategies developed for the correlation market
- CDO tranches on standard Index such as CDX North America Investment Grade Index



Introduction

Several risks at hand which may sometimes overlap:

- Default risk of reference entities
  - Cash-flows of synthetic CDO tranches are driven by the evolution of the portfolio loss

$$L_t = \frac{1}{n} \sum_{i=1}^{n} (1 - R_i) \mathbf{1}_{\{\tau_i \leq t\}}$$

- Correlation risk
- Credit spread risk or Market risk
  - Evolution of market prices after inception
- Contagion risk
  - Dynamic combination of credit spread risk and default risk

Introduction

- Credit crisis has deeply affected the market of CDS index tranches
  - Series 10 of CDX.NA.IG suffers defaults of Fannie Mae, Freddie Mac and Lehman Brothers
  - High level of credit spreads and volatility
- Recent revision of Basel II regulation concerns risk-management of credit derivatives
  - Residual risks resulting from dynamic hedging strategies must be reflected in the capital charge
- Performance and efficiency of underlying hedging methods is a topical issue

Hedging loss derivatives

Generally speaking, ...

Hedging derivative instruments consists in taking opposite positions in some primary liquid instruments whose market values are sensitive to the same underlying risks

- The aim is to minimize the overall exposure to market price evolution
- Composition of the hedging portfolio need to be regularly updated over time
- Require a pricing device to compute hedging strategies

Hedging loss derivatives

In this study, ...

- Hedging of a buy or sell protection position on an index CDO tranche
- Hedging portfolio composed of two instruments:
  - CDS Index
  - Savings account

Performance analysis of alternative hedging methods:

- Δ^{Gauss}: delta of the tranche computed within the one-factor Gaussian copula model (standard quotation device)
- Δ^{lo}: delta of the tranche computed within the local intensity model (two specifications of model parameters)

Hedging loss derivatives

Gauss delta:

$$\Delta_t^{\text{Gauss}} = \frac{\mathcal{V}(t, S_t + \varepsilon, \rho_t) - \mathcal{V}(t, S_t, \rho_t)}{\mathcal{V}^I(t, S_t + \varepsilon) - \mathcal{V}^I(t, S_t)}$$

- $\mathcal{V}$: price of the tranche computed in the Gaussian copula model
- $\mathcal{V}^I$: price of the CDX index computed in the Gaussian copula model
- S_t: credit spread of the CDS index at time t
- $\varepsilon = 1$ bp
- ρ_t: implied correlation parameter of the tranche at time t

Gauss delta = Sensitivity with respect to the CDS Index spread using the industry standard quotation device

Hedging loss derivatives

Local intensity delta:

$$\Delta_t^{\text{lo}} = \frac{V\left(t, N_t + 1\right) - V\left(t, N_t\right)}{V^I\left(t, N_t + 1\right) - V^I\left(t, N_t\right)}.$$

- V: price of the tranche computed in the local intensity model
- V^I: price of the CDX index computed in the local intensity model
- N_t: current number of defaults

Local intensity delta = Jump-to-Default delta computed using the local intensity model

Local intensity model

- N_t is a continuous-time Markov chain (pure birth process) with intensity matrix:

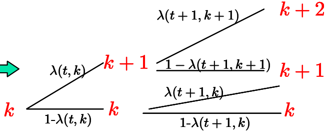
$$\Lambda(t) = \begin{pmatrix} -\lambda(t,0) & \lambda(t,0) & 0 & & 0 \\ 0 & -\lambda(t,1) & \lambda(t,1) & & 0 \\ & & \ddots & \ddots & \\ 0 & & & -\lambda(t,n-1) & \lambda(t,n-1) \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Model involves as many parameters as the number of names
- This parallels the Dupire's local volatility approach developed for the equity derivative market

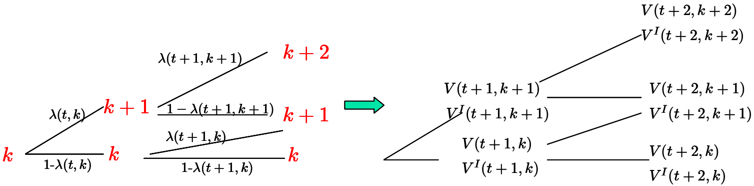
Local intensity model

- Binomial tree: discrete version of the local intensity model

$$\Lambda(t) = \begin{pmatrix} -\lambda(t,0) & \lambda(t,0) & 0 & & 0 \\ 0 & -\lambda(t,1) & \lambda(t,1) & & 0 \\ & & \ddots & \ddots & \\ 0 & & & -\lambda(t,n-1) & \lambda(t,n-1) \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



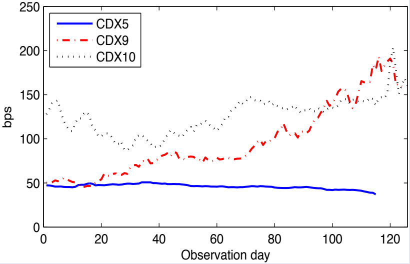
- Given some loss intensities $\lambda(t,k)$, CDO tranches and index prices computed by backward induction:



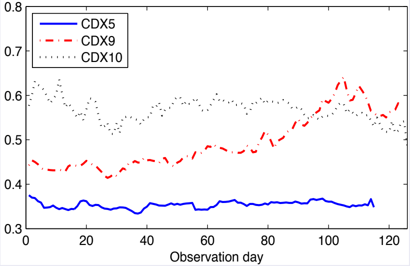
Data set

- 5-year CDX NA IG Series 5 from 20 September 2005 to 20 March 2006
- 5-year CDX NA IG Series 9 from 20 September 2007 to 20 March 2008
- 5-year CDX NA IG Series 10 from 21 March 2008 to 20 September 2008

Index spreads



Base correlation at 3% strike



Empirical results

Calibration results of model parameters in the three approaches:

- **Gauss**: Gaussian copula model with one implied correlation parameter per standard tranche (base correlation approach)
- **Para**: Local intensity model – parametric specification of local itensities

$$\lambda(t,k) = \lambda(k) = (n-k)\sum_{i=0}^{k} b_i$$

(Herbertsson (2008))

- **EM**: Local intensity model – local itensities $\lambda(t,k)$ obtained by minimizing a relative entropy distance with respect to a prior distribution

$$\inf_{\mathbb{Q}\in\Lambda} \mathbb{E}^{\mathbb{Q}_0}\left[\frac{d\mathbb{Q}}{d\mathbb{Q}_0}\ln\left(\frac{d\mathbb{Q}}{d\mathbb{Q}_0}\right)\right]$$

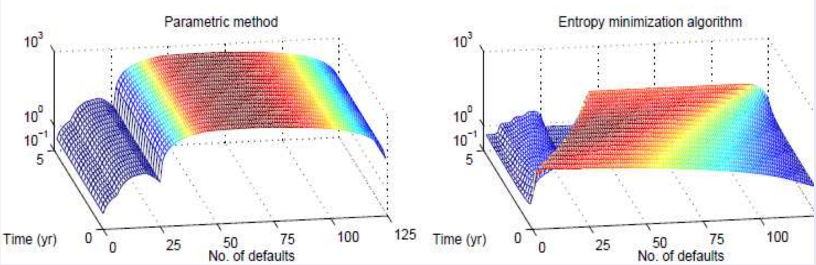
(Cont and Minca (2008))

Empirical results

Root mean squared calibration errors (in percentage):

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
Index	0.04	5.15	5.14	0.03	4.40	4.81	0.02	6.73	6.77
0%-3%	0.01	2.35	2.36	0.00	1.31	1.32	0.01	1.69	1.68
3%-7%	0.00	0.51	0.69	0.00	0.61	0.86	0.00	1.04	1.03
7%-10%	0.00	0.08	1.32	0.00	0.24	0.91	0.00	0.43	0.39
10%-15%	0.00	0.06	1.77	0.00	0.24	1.15	0.00	0.40	0.36
15%-30%	0.00	0.29	1.97	0.01	1.19	1.74	0.01	1.80	1.68

Comparison of typical shapes of local intensities $\lambda(t,k)$, Para (left), EM (right)



Empirical results

Comparison of three alternative hedging methods

- Gauss delta: index Spread sensitivity computed in a one-factor Gaussian copula model

$$\Delta_t^{\text{Gauss}} = \frac{\mathcal{V}(t, S_t + \varepsilon, \rho_t) - \mathcal{V}(t, S_t, \rho_t)}{\mathcal{V}^I(t, S_t + \varepsilon) - \mathcal{V}^I(t, S_t)}$$

  where $\mathcal{V}$ and $\mathcal{V}^I$ are the Gaussian copula pricing function associated with (resp.) the tranche and the CDS index.

- Local intensity delta:

$$\Delta_t^{\text{lo}} = \frac{V(t, N_t + 1) - V(t, N_t)}{V^I(t, N_t + 1) - V^I(t, N_t)}.$$

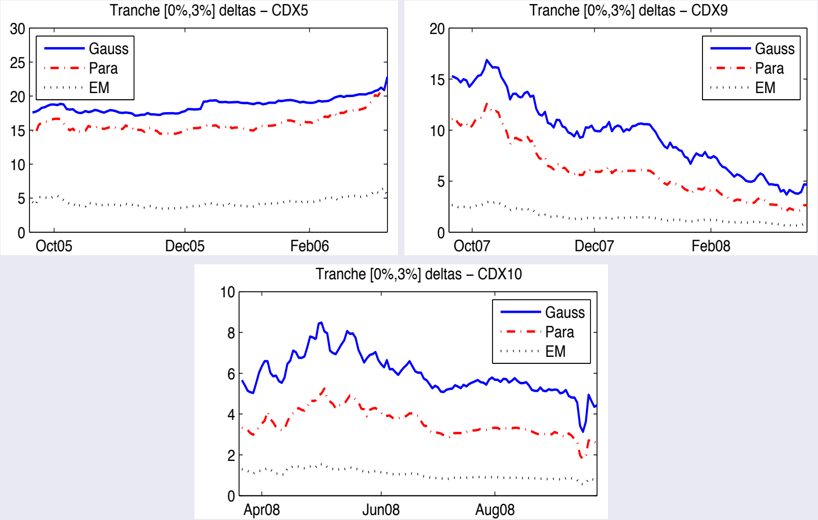
  with both Parametric (Para) and Entropy Minimisation (EM) calibration methods

Credit deltas on 20 September 2007 (normalized to tranche notional)

Tranche	Gauss	Para	EM
0%-3%	15.29	11.05	2.64
3%-7%	5.03	4.59	2.70
7%-10%	1.94	2.26	2.29
10%-15%	1.10	1.47	1.99
15%-30%	0.60	1.01	1.74

Empirical results

Time series of equity tranche credit deltas, CDX.NA.IG series 5, 9 and 10



Hedging performance

Back-testing hedging experiments on series 5, 9 and 10

- Hedging portfolio rebalanced everyday (dt=1) or every 5 days (dt=5)
- P&L (Profit-and-Loss) increment of hedged position:

$$\delta P\&L(t) = \delta V_m(t) - \Delta_t \cdot \delta V_m^I(t)$$

  - $\delta V_m(t) = V_m(t+dt) - V_m(t)$: Increment of tranche market value
  - $\delta V_m^I(t) = V_m^I(t+dt) - V_m^I(t)$: Increment of index market value
  - Δ_t: One of the previous hedging ratios computed at time t

- P&L increments evaluated in the same frequency as rebalancing

Hedging performance

Two metrics to compare the hedging strategies:

$$\textbf{Relative hedging error} = \left| \frac{\text{Average P\&L increment of the hedged position}}{\text{Average P\&L increment of the unhedged position}} \right| = \left| \frac{\text{Average of } \delta P\&L(t)}{\text{Average of } \delta V_m(t)} \right|$$

$$\textbf{Residual volatility} = \frac{\text{P\&L increment volatility of the hedged position}}{\text{P\&L increment volatility of the unhedged position}} = \frac{\text{Volatility of } \delta P\&L(t)}{\text{Volatility of } \delta V_m(t)}$$

Hedging performance for 1-day rebalancing

Relative hedging errors (in percentage)

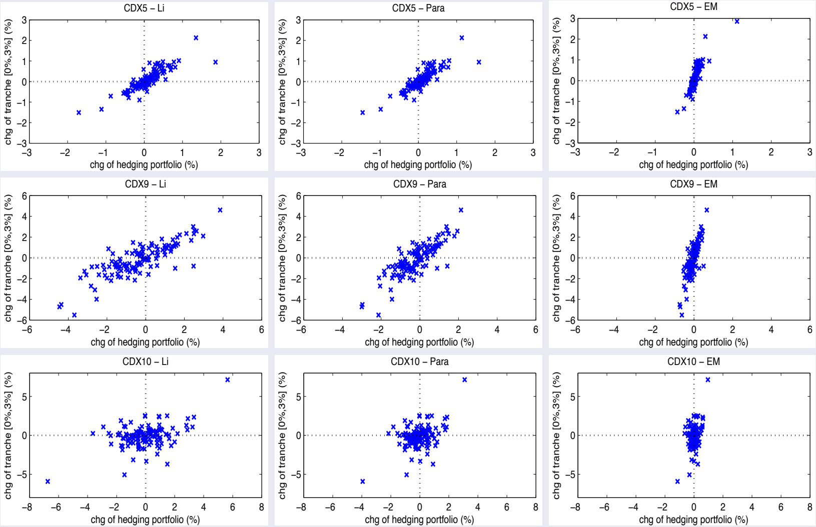
	CDX5			CDX9			CDX10		
Tranche	Li	Para	EM	Li	Para	EM	Li	Para	EM
0%-3%	4	5	73	80	10	72	33	55	90
3%-7%	1	3	35	0.4	19	59	48	49	75
7%-10%	10	10	43	15	13	37	49	25	44
10%-15%	7	27	131	27	18	14	139	181	208
15%-30%	0.54	61	324	3	32	89	172	269	396

Residual volatilities (in percentage)

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
0%-3%	45	47	79	59	59	87	105	91	93
3%-7%	70	72	68	58	47	64	85	74	78
7%-10%	90	101	120	53	50	46	83	79	70
10%-15%	90	107	188	61	63	60	91	93	86
15%-30%	93	110	256	37	49	77	84	99	127

Detailed results for the [0–3%]-equity tranche

Daily changes of equity tranche value against daily changes of hedging portfolio value



Hedging performance for 5-day rebalancing

Relative hedging errors (in percentage)

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
0%-3%	6	10	77	59	2	73	24	48	88
3%-7%	16	16	51	2	18	58	48	43	72
7%-10%	19	1	15	11	12	36	50	15	41
10%-15%	22	8	75	13	5	5	141	198	209
15%-30%	21	30	207	1	35	86	127	227	382

Residual volatilities (in percentage)

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
0%-3%	42	46	83	50	56	86	71	72	89
3%-7%	75	75	66	73	65	71	43	40	64
7%-10%	99	118	135	57	56	54	40	38	44
10%-15%	82	110	202	94	98	95	42	44	40
15%-30%	77	108	298	46	69	108	31	33	54

Conclusion

- All model specifications perfectly fit CDO tranche quotes
- However, for the local intensity model, the two introduced specifications give strikingly different deltas and dramatically different hedging performances
- Hedging based on local intensity model with Entropy Minimisation calibration gives poor performance
- No clear evidence to distinguish the performance of hedging based on the Gaussian copula model and on the parametric local intensity model