Dynamic Hedging of Portfolio Credit Risk in a Markov Copula Model

Areski Cousin
ISFA, Université Lyon 1

International Workshop on Finance 2012

Tokyo, 30–31 October 2012

Introduction

Tom Bielecki, Areski Cousin, Stéphane Crépey and Alexander Herbertsson (2012a)

Dynamic Modeling of Portfolio Credit Risk with Common Shocks

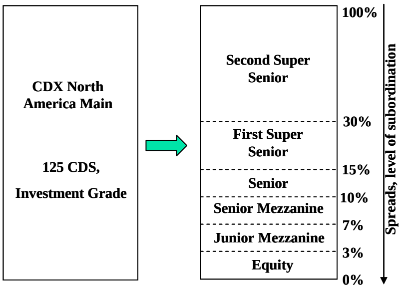
Tom Bielecki, Areski Cousin, Stéphane Crépey and Alexander Herbertsson (2012b)

A Markov Copula Model of Portfolio Credit Risk with Stochastic Intensities and Random Recoveries

Available on defaultrisk.com

Introduction

Main issue: hedging of portfolio credit derivatives



- Cash-flows driven by the realized path of the aggregate loss process

$$L_t = \frac{1}{n} \sum_{i=1}^{n} (1 - R_i) H_t^i$$

where R_i is the recovery rate and H_t^i is the default indicator of obligor i

Introduction

In this paper, we construct a bottom-up dynamic model consisting of

- $\mathbf{X} = (X^1, \ldots, X^n)$ a vector of factor processes driving credit spreads
- $\mathbf{H} = (H^1, \ldots, H^n)$ a vector of default processes ($H_t^i = 1$ iif default of name i occurs before time t)
- $\mathcal{F}_t = \mathcal{F}_t^{\mathbf{X},\mathbf{H}}$

and with the following key features :

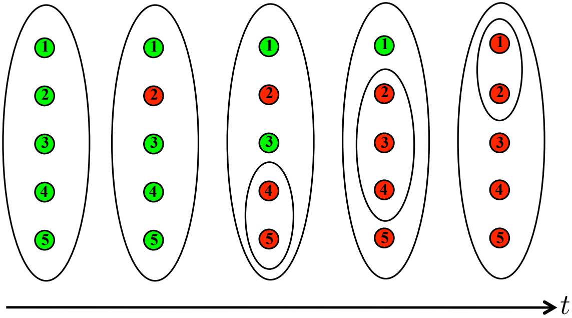
- Markovian model: $(\mathbf{X}, \mathbf{H})$ is a Markov process with respect to $\mathcal{F}$
- Markov copula property: Each pair (X^i, H^i) is a Markov process with respect to $\mathcal{F}$
- Tractable model: Computation of CDS spreads, CDO tranche prices and hedging strategies can be achieved by fast numerical procedure

Markov copula model of portfolio credit risk

Simultaneous default model

- Defaults are the consequence of triggering-events affecting pre-specified groups of obligors

Example: $n = 5$ and $\mathcal{Y} = \{\{1\},\{2\},\{3\},\{4\},\{5\},\{4,5\},\{2,3,4\},\{1,2\}\}$.



Markov copula model of portfolio credit risk

- $\{1, \ldots, n\}$ set of credit references
- $\mathcal{Y} = \{\{1\}, \ldots, \{n\}, I_1, \ldots, I_m\}$ pre-specified groups of obligors
- $\lambda_Y = \lambda_Y(t)$ deterministic intensity function of the triggering-event associated with group $Y \in \mathcal{Y}$
- $\mathbf{H}_t = (H_t^1, \ldots, H_t^n)$ defined as an n-dimensional Markov chain in $\{0,1\}^n$ such that for $\mathbf{k}, \mathbf{m} \in \{0,1\}^n$:

$$\mathbb{P}\left(\mathbf{H}_{t+dt} = \mathbf{m} \mid \mathbf{H}_t = \mathbf{k}\right) = \sum_{Y \in \mathcal{Y}} \lambda_Y(t) \mathbf{1}_{\{\mathbf{k}^Y = \mathbf{m}\}} dt$$

  where $\mathbf{k}^Y$ is obtained from $\mathbf{k} = (k_1, \ldots, k_n)$ by replacing the components k_j, $j \in Y$, by number one. ex: $(0,1,0,0)^{\{1,2,4\}} = (1,1,0,1)$
- $\mathcal{F}_t = \sigma(\mathbf{H}_u \,, u \leq t)$ natural filtration of $\mathbf{H}$

Markov copula model of portfolio credit risk

Example: Portfolio with $n = 2$ names

$\mathcal{Y} = \{\{1\}, \{2\}, \{1,2\}\}$. $\mathbf{H}_t = (H_t^1, H_t^2)$ is a bivariate continuous-time Markov chain with space set $\{(0,0), (1,0), (0,1), (1,1)\}$ and generator matrix

$$\begin{array}{c} \\ (0,0) \\ (1,0) \\ (0,1) \\ (1,1) \end{array} \begin{array}{c} \begin{array}{cccc} (0,0) & (1,0) & (0,1) & (1,1) \end{array} \\ \left(\begin{array}{cccc} - & \lambda_{\{1\}} & \lambda_{\{2\}} & \lambda_{\{1,2\}} \\ 0 & - & 0 & \lambda_{\{2\}} + \lambda_{\{1,2\}} \\ 0 & 0 & - & \lambda_{\{1\}} + \lambda_{\{1,2\}} \\ 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

- Obligor 1 defaults with intensity $\lambda_{\{1\}} + \lambda_{\{1,2\}}$ regardless of the state of the pool
- Obligor 2 defaults with intensity $\lambda_{\{2\}} + \lambda_{\{1,2\}}$ regardless of the state of the pool

Markov copula model of portfolio credit risk

n-dimensional case

Obligor i defaults with intensity

$$\lambda_i(t) \quad = \quad \lambda_{\{i\}}(t) + \sum_{k=1}^{m} \lambda_{I_k}(t) \mathbf{1}_{\{i \in I_k\}}$$

Then,

$$\mathbb{P}(H^i_{t+dt} - H^i_t = 1 \mid \mathcal{F}_t) = \mathbb{P}(H^i_{t+dt} - H^i_t = 1 \mid H^i_t) = (1 - H^i_t)\lambda_i(t)dt$$

- Each default indicator process H^i, $i = 1, \ldots, n$ is Markov with respect to $\mathcal{F}$: strong Markov copula property (Bielecki, Vidozzi and Vidozzi 2008)
- On economic grounds, this means that there is no contagion effect : past defaults do not have any effect on intensities of surviving names

Markov copula model of portfolio credit risk

The previous construction can be extended to the case of stochastic intensity functions:

$$\lambda_Y = \lambda_Y(t, X_t^Y)\,,\; Y \in \mathcal{Y}$$

where $\mathbf{X} = (X^Y)_{Y \in \mathcal{Y}}$ is a strong solution to

$$dX_t^Y = b_Y(t, X_t^Y)\,dt + \sigma_Y(t, X_t^Y)\,dW_t^Y\,, \quad (1)$$

with

- $b_Y(t, x)$ and $\sigma_Y(t, x)$ are suitable drift and diffusion functions
- W^Y, $Y \in \mathcal{Y}$ are independent standard brownian motions

Markov copula model of portfolio credit risk

Markov property of the model

Let $\mathcal{F} = \mathcal{F}^{\mathbf{X},\mathbf{H}}$ be the natural filtration of $(\mathbf{X}, \mathbf{H})$. The process $(\mathbf{X}, \mathbf{H})$ is an $\mathcal{F}$-Markov process with generator $\mathcal{A}$ given by

$$\mathcal{A}_t u(t,\mathbf{x},\mathbf{k}) = \sum_{Y \in \mathcal{Y}} \left(b_Y(t,x_Y)\partial_{x_Y} u(t,\mathbf{x},\mathbf{k}) + \frac{1}{2}\sigma_Y^2(t,x_Y)\partial^2_{x_Y^2} u(t,\mathbf{x},\mathbf{k}) \right) + \sum_{Y \in \mathcal{Y}} \lambda_Y(t,x_Y)\delta u^Y(t,\mathbf{x},\mathbf{k})$$

Could we find $\mathbf{X}$ and intensity functions $\lambda_Y(\cdot,\cdot)$, $Y \in \mathcal{Y}$ such that the Markov copula property holds ?

- i.e., for every $i = 1, \ldots, n$, the process (X^i, H^i) is an $\mathcal{F}$-Markov process ?

Possible solution : generalized CIR intensities

$$\lambda_Y(t, \mathbf{X}_t) = X_t^Y \ , \ Y \in \mathcal{Y}$$

where X_t^Y, $Y \in \mathcal{Y}$ are generalized CIR processes such that

$$dX_t^Y = a\left(b_Y(t) - X_t^Y\right)dt + c\sqrt{X_t^Y}dW_t^Y$$

- a and c are positive parameters common to all groups
- $b_Y(t)$ is specific to group Y and defined by a deterministic function of time
- W^Y, $Y \in \mathcal{Y}$ are independent brownian motions

Markov copula model of portfolio credit risk

Possible solution : generalized CIR intensities (cont.)

In this framework, the default intensity of name i is given by

$$X_t^i = X_t^{\{i\}} + \sum_{k=1}^{m} X_t^{I_k} \mathbf{1}_{\{i \in I_k\}}$$

Note that X^i is again a generalized CIR process such that

$$dX_t^i = a\left(b_i(t) - X_t^i\right) dt + c\sqrt{X_t^i} dW_t^i$$

where

- $b_i(t) = b_{\{i\}}(t) + \sum_{k=1}^{m} b_{I_k}(t) \mathbf{1}_{\{i \in I_k\}}$
- W^i is a standard brownian motion

Markov copula model of portfolio credit risk

Markov copula property for extended CIR intensities (Bielecki et al. (2012a))

Let $\mathcal{F} = \mathcal{F}^{\mathbf{X},\mathbf{H}}$ be the natural filtration of $(\mathbf{X}, \mathbf{H})$ where $\mathbf{X} = (X^Y)_{Y \in \mathcal{Y}}$ are previously defined CIR processes. Then,

- $(\mathbf{X}, \mathbf{H})$ is an $\mathcal{F}$-Markov process
- For every $i = 1, \ldots, n$, the process (X^i, H^i) is an $\mathcal{F}$-Markov process

Moreover, the $\mathcal{F}_t$–conditional survival probability of name i is given by

$$\mathbb{P}(\tau_i > T \,|\, \mathcal{F}_t) = (1 - H^i_t)\mathbb{E}\left\{\exp\left(-\int_t^T X^i_u du\right) \,|\, X^i_t\right\}$$

- Analytical expression available in the case of piecewise-constant $b^Y(\cdot)$ (See Bielecki et al. (2012b))
- Other stable-by-convolution processes can be considered (e.g., processes driving by Levy-subordinators)

Common-Shock Model Interpretation

Main result: equivalent $\mathcal{F}_t$–related common-shocks model

- $\mathcal{Y}_t = \{Y \in \mathcal{Y}\,;Y \not\subseteq \text{supp}(\mathbf{H}_t)\}$: set of pre-specified groups that contain at least one alive obligor
- For any pre-specified group $Y \in \mathcal{Y}_t$, we define

$$\tau_Y(t) = \inf\left\{\theta \geq t \mid \int_t^\theta \lambda_Y(s, \mathbf{X}_s)ds > E_Y\right\}$$

  where E_Y, $Y \in \mathcal{Y}_t$, are independent and exponentially distributed random variables with parameter 1.

- In the $\mathcal{F}_t$–related common-shock model, the individual default time of a non-defaulted name i is defined by:

$$\widehat{\tau}_i(t) = \min_{\{Y \in \mathcal{Y}_t\,;\, i \in Y\}} \tau_Y(t)$$

- $H^i_\theta(t) = \mathbf{1}_{\{\widehat{\tau}_i(t) \leq \theta\}}$: default indicator of name i at time θ in the $\mathcal{F}_t$–related common-shock model

Common-Shock Model Interpretation

Main result

Proposition

Let Z be a subset of $\{1, \ldots, n\}$. For every $\theta_1, \ldots, \theta_n \geq t$, one has on the event $\{Z = \mathrm{supp}^c(\mathbf{H}_t)\}$:

$$\mathbb{P}\left(\tau_i > \theta_i,\, i \in \mathrm{supp}^c(\mathbf{H}_t) \mid \mathcal{F}_t\right) = \mathbb{P}\left(\widehat{\tau}_i(t) > \theta_i,\, i \in Z \mid \mathbf{X}_t\right)$$

Moreover, if

- $N_\theta = \sum_{i=1}^n H_\theta^i$ denotes the cumulative number of defaults at time θ in the Markov model
- $N_\theta(t, Z) = n - |Z| + \sum_{i \in Z}^n H_\theta^i(t)$ denotes the cumulative number of defaults at time θ in the $\mathcal{F}_t$-related common-shock model

then, for every $\theta \geq t$, one has on the event $\{Z = \mathrm{supp}^c(\mathbf{H}_t)\}$:

$$\mathbb{P}\left(N_\theta = k \mid \mathcal{F}_t\right) = \mathbb{P}\left(N_\theta(t, Z) = k \mid \mathbf{X}_t\right)$$

for any $k = n - |Z|, \ldots, n$.

Common-Shock Model Interpretation

- But, for any time $\theta \geq t$, $H^i_\theta(t)$, $i \in Z$, are conditionally independent Bernoulli's given $\left(H^{I_1}_\theta(t), \ldots, H^{I_m}_\theta(t)\right)$
- Fast convolution-recursion procedure can be used to compute marginal loss distributions conditionally on any given set $\{Z = \text{supp}^c(\mathbf{H}_t)\}$
- As far as standard CDO tranches are concerned, we will see that pricing, calibration and computation of hedging strategies are numerically tractable

Hedging CDO tranches with single-name CDS-s

Set of fundamental martingales for jump components of the Markov model

- H_t^Z is the indicator process of simultaneous default of names in the set Z, for every subset Z of $\{1, \ldots, n\}$
- $Y_t = Y \cap \mathrm{supp}^c(\mathbf{H}_{t-})$ stands for the set of survivors of set Y right before t, for every pre-specified group $Y \in \mathcal{Y}$

Set of fundamental martingales

The process M^Z defined by

$$dM_t^Z := dH_t^Z - \ell_Z(t, \mathbf{X}_t, \mathbf{H}_{t-})dt$$

is a martingale with respect to $\mathcal{F}$, where the intensity function $\ell_Z(t, \mathbf{x}, \mathbf{k})$ is such that

$$\ell_Z(t, \mathbf{X}_t, \mathbf{H}_{t-}) = \sum_{Y \in \mathcal{Y};\, Y_t = Z} \lambda_Y(t, \mathbf{X}_t)$$

Hedging CDO tranches with single-name CDS-s

Itô formula

Given a "regular enough" function $u = u(t, \mathbf{x}, \mathbf{k})$, one has, for $t \in [0, T]$,

$$du(t, \mathbf{X}_t, \mathbf{H}_t) = \Big(\partial_t + \mathcal{A}_t\Big) u(t, \mathbf{X}_t, \mathbf{H}_t) dt + \nabla u(t, \mathbf{X}_t, \mathbf{H}_t) \sigma(t, \mathbf{X}_t) d\mathbf{W}_t + \sum_{Z \in \mathcal{Z}_t} \delta u^Z(t, \mathbf{X}_t, \mathbf{H}_{t-}) dM_t^Z$$

where

- $\sigma(t, \mathbf{x})$: diagonal matrix with diagonal $(\sigma_Y(t, x_Y))_{Y \in \mathcal{Y}}$
- $\nabla u(t, \mathbf{x}, \mathbf{k}) = (\partial_{x_1} u(t, \mathbf{x}, \mathbf{k}), \ldots, \partial_{x_\nu} u(t, \mathbf{x}, \mathbf{k}))$
- $\delta u^Z(t, \mathbf{x}, \mathbf{k}) = u(t, \mathbf{x}, \mathbf{k}^Z) - u(t, \mathbf{x}, \mathbf{k})$
- $\mathcal{Z}_t = \{Y_t;\, Y \in \mathcal{Y}\} \setminus \emptyset$: set of all non-empty sets of survivors of sets Y in $\mathcal{Y}$ right before time t

Martingale dimension: $n + m + 2^n$

Hedging CDO tranches with single-name CDS-s

Price dynamics for single-name CDS-s (buy-protection position)

- T: contract maturity
- S_i: T-year contractual CDS-spread of obligor i
- $t_1 < \cdots < t_p = T$: premium payment dates, $h = t_j - t_{j-1}$ length between two premium payment dates (typically a quarter)
- R_i: recovery rate of obligor i

Except for numerical results, we will assume zero interest rates

Hedging CDO tranches with single-name CDS-s

Price dynamics for single-name CDS i

The price P^i and the cumulative value $\widehat{P}^i$ at time $t \in [0, T]$ of a single-name CDS on obligor i are given by

$$\begin{aligned} P_t^i &= \mathbf{1}_{\{\tau_i > t\}} v_i(t, X_t^i) \\ d\widehat{P}_t^i &= \mathbf{1}_{\{\tau_i > t\}} \partial_{x_i} v_i(t, X_t^i) \sigma_i(t, X_t^i) dW_t^i \\ &\quad + \sum_{Z \in \mathcal{Z}_t} \mathbf{1}_{\{i \in Z\}} \left(1 - R_i - v_i(t, X_t^i)\right) dM_t^Z \end{aligned}$$

for a pre-default pricing function $v_i(t, x_i)$ such that

$$\mathbf{1}_{\{\tau_i > t\}} v_i(t, X_t^i) = \mathbb{E}[(1 - R_i)\mathbf{1}_{\{t < \tau_i \le T\}} - S_i h \sum_{t < t_j \le T} \mathbf{1}_{\{\tau_i > t_j\}} | \mathcal{F}_t]$$

Hedging CDO tranches with single-name CDS-s

Price dynamics for CDO tranche $[a, b]$ (buy-protection position)

- T: contract maturity
- a: attachement point, b: detachement point, $0 \leq a < b \leq 1$
- $S^{a,b}$: T-year contractual spread of CDO tranche $[a, b]$
- $t_1 < \cdots < t_p = T$: premium payment dates, $h = t_j - t_{j-1}$ length between two premium payment dates (typically a quarter)
- CDO tranche cash-flows are driven by the tranche loss process

$$L_t^{a,b} = L_{a,b}(\mathbf{H}_t) = (L_t - a)^+ - (L_t - b)^+$$

where

$$L_t = L_t(\mathbf{H}_t) = \frac{1}{n} \sum_{i=1}^{n} (1 - R_i) H_t^i$$

is the credit loss process for the underlying portfolio

Hedging CDO tranches with single-name CDS-s

Price dynamics for CDO tranche $[a, b]$

The price Π and the cumulative value $\widehat{\Pi}$ at time $t \in [0, T]$ of a CDO-tranche $[a, b]$ are given by

$$\Pi_t = u(t, \mathbf{X}_t, \mathbf{H}_t)$$

$$d\widehat{\Pi}_t = \nabla u(t, \mathbf{X}_t, \mathbf{H}_t)\sigma(t, \mathbf{X}_t)d\mathbf{W}_t + \sum_{Z \in \mathcal{Z}_t} \Big(L_{a,b}(\mathbf{H}^Z_{t-}) - L_{a,b}(\mathbf{H}_{t-}) + \delta u^Z(t, \mathbf{X}_t, \mathbf{H}_{t-}) \Big)\, dM^Z_t$$

for a pricing function $u(t, \mathbf{x}, \mathbf{k})$ such that

$$u(t, \mathbf{X}_t, \mathbf{H}_t) = \mathbb{E}\Big[L^{a,b}_T - L^{a,b}_t - S^{a,b}\, h \sum_{t < t_j \le T} \Big(b - a - L^{a,b}_{t_j} \Big) \Big| \mathcal{F}_t \Big]$$

The pricing function $u(t, \mathrm{x}, \mathrm{k})$ solves a very large system of Kolmogorov pde. Thanks to the common-shock interpretation, it can be computed by fast recursion procedures.

Hedging CDO tranches with single-name CDS-s

Hedging portfolio: first d single-name CDS-s and the savings account

The vector of cumulative values $\widehat{\mathbf{P}} = (\widehat{P}^1, \ldots, \widehat{P}^d)^\mathsf{T}$ associated with the first d CDS-s has the following dynamics:

$$d\widehat{\mathbf{P}}_t = \nabla \mathbf{v}(t, \mathbf{X}_t, \mathbf{H}_t)\sigma(t, \mathbf{X}_t)d\mathbf{W}_t + \sum_{Z \in \mathcal{Z}_t} \Delta \mathbf{v}^Z(t, \mathbf{X}_t, \mathbf{H}_{t-})dM_t^Z$$

where

- $\nabla \mathbf{v}$ is a $d \times \nu$-matrix such that $\nabla \mathbf{v}(t, \mathbf{x}, \mathbf{k})_i^j = \mathbf{1}_{\{k_j=0\}} \partial_{x_j} v_i(t, x_i)$, for every $1 \le i \le d$ and $1 \le j \le \nu$

- $\Delta \mathbf{v}^Z(t, \mathbf{x}, \mathbf{k})$ is a d-dimensional column vector equal to

$$\left(\mathbf{1}_{\{1 \in Z,\, k_1=0\}} \left((1 - R_1) - v_1(t, x_1)\right), \ldots, \mathbf{1}_{\{d \in Z,\, k_d=0\}} \left((1 - R_d) - v_d(t, x_d)\right)\right)^\mathsf{T}$$

Hedging CDO tranches with single-name CDS-s

Tracking error: Process (e_t) such that $e_0 = 0$ and for $t \in [0, T]$:

$$
\begin{aligned}
de_t &= d\widehat{\Pi}_t - \zeta_t d\widehat{\mathbf{P}}_t \\
&= \Big(\nabla u(t, \mathbf{X}_t, \mathbf{H}_t) - \zeta_t \nabla \mathbf{v}(t, \mathbf{X}_t, \mathbf{H}_t)\Big)\sigma(t, \mathbf{X}_t) d\mathbf{W}_t \\
&\quad + \sum_{Z \in \mathcal{Z}_t} \Big(\Delta u^Z(t, \mathbf{X}_t, \mathbf{H}_{t-}) - \zeta_t \Delta \mathbf{v}^Z(t, \mathbf{X}_t, \mathbf{H}_{t-})\Big) dM_t^Z
\end{aligned}
$$

where

- $\zeta_t = (\zeta_t^1, \ldots, \zeta_t^d)$ gives the positions held at time t in CDS $1, \ldots, d$
- $\nabla u(t, \mathbf{x}, \mathbf{k}) = (\partial_{x_1} u(t, \mathbf{x}, \mathbf{k}), \ldots, \partial_{x_\nu} u(t, \mathbf{x}, \mathbf{k}))$
- $\Delta u^Z(t, \mathbf{x}, \mathbf{k}) = \delta^Z u(t, \mathbf{x}, \mathbf{k}) + L_{a,b}(\mathbf{k}^Z) - L_{a,b}(\mathbf{k})$

Hedging CDO tranches with single-name CDS-s

Min-variance hedging strategies

The min-variance hedging strategy ζ for the CDO-tranche $[a, b]$ is

$$\zeta_t = \frac{d\langle \widehat{\Pi}, \widehat{\mathbf{P}} \rangle_t}{dt} \left(\frac{d\langle \widehat{\mathbf{P}} \rangle_t}{dt} \right)^{-1} = \zeta(t, \mathbf{X}_t, \mathbf{H}_{t-})$$

where $\zeta = (u, \mathbf{v})(\mathbf{v}, \mathbf{v})^{-1}$, with

$$(u, \mathbf{v}) = (\nabla u)\sigma^2(\nabla \mathbf{v})^{\mathsf{T}} + \sum_{Y \in \mathcal{Y}} \lambda_Y \Delta u^Y (\Delta \mathbf{v}^Y)^{\mathsf{T}}$$

$$(\mathbf{v}, \mathbf{v}) = (\nabla \mathbf{v})\sigma^2(\nabla \mathbf{v})^{\mathsf{T}} + \sum_{Y \in \mathcal{Y}} \lambda_Y \Delta \mathbf{v}^Y (\Delta \mathbf{v}^Y)^{\mathsf{T}}$$

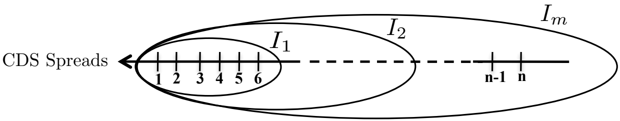
Calibration on CDX index

Data set: 5-year CDX North-America IG index on 20 December 2007

- Quoted spreads at different pillars of the $n = 125$ index constituents
- Quoted spreads of standard tranches [0,3], [3,7], [7,10], [10,15], [15,30]

Model specification:

- Names are labelled with respect to decreasing level of spreads



- $m = 5$ groups $I_1 \subset \cdots \subset I_5$ such that $I_1 = \{1, \ldots, 6\}$, $I_2 = \{1, \ldots, 19\}$, $I_3 = \{1, \ldots, 25\}$, $I_4 = \{1, \ldots, 61\}$, $I_5 = \{1, \ldots, 125\}$
- Piecewise-constant intensities $\lambda_{\{1\}}, \ldots, \lambda_{\{125\}}$, $\lambda_{I_1}, \ldots, \lambda_{I_5}$ with grid points corresponding to CDS pillars
- Homogeneous and constant recovery rates: 40%
- Constant short-term interest rate: 3%

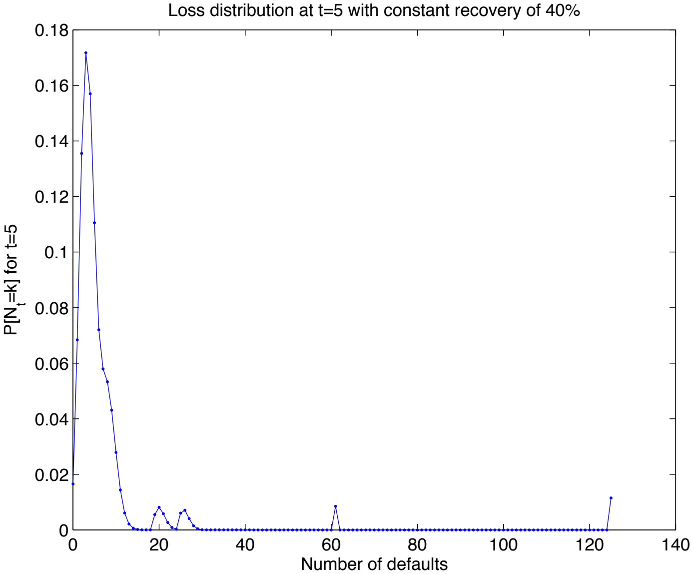
Calibration on CDX index

Calibration results:

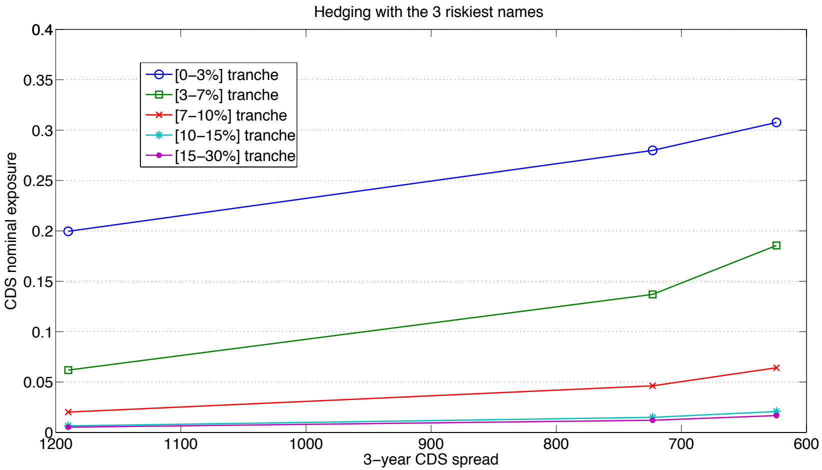
Tranche	[0,3]	[3,7]	[7,10]	[10,15]	[15,30]
Model spread in bps	48.0701	254.0000	124.0000	61.0000	38.9390
Market spread in bps	48.0700	254.0000	124.0000	61.0000	41.0000
Abs. Err. in bps	0.0001	0.0000	0.0000	0.0000	2.0610
% Rel. Err.	0.0001	0.0000	0.0000	0.0000	5.0269

- Names in the set $I_5 \setminus I_4$ are excluded from the calibration constraints (they can only default within the Armageddon shock I_5)

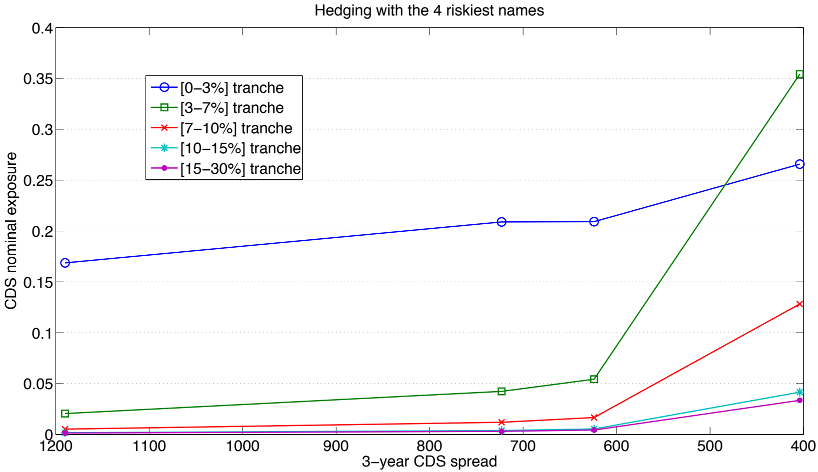
Implied 5-year loss distribution:



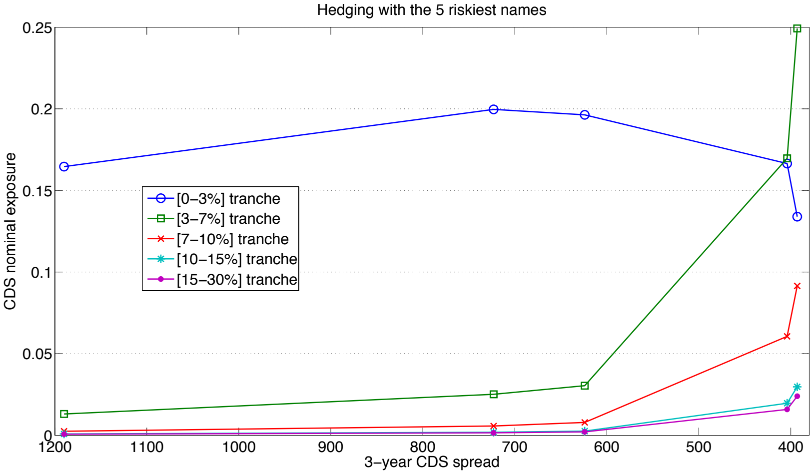
Min-variance hedging strategies



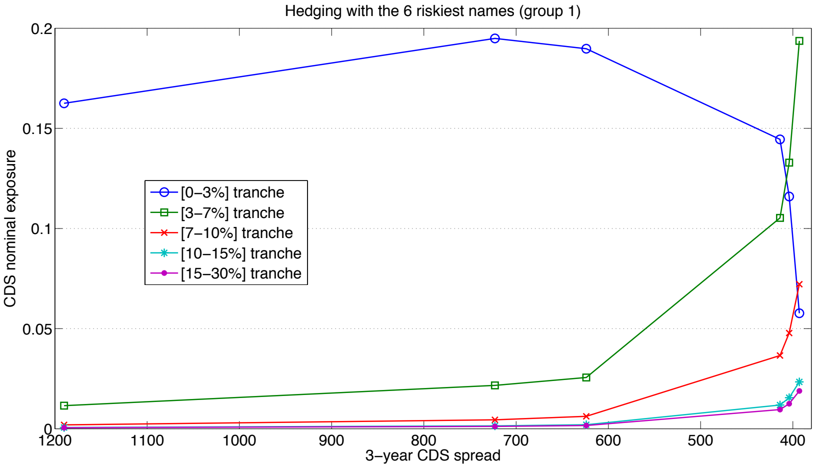
Min-variance hedging strategies



Min-variance hedging strategies

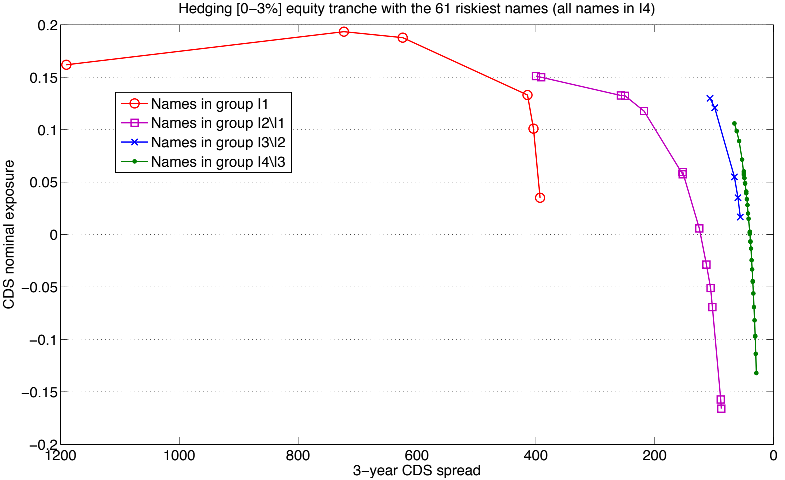


Min-variance hedging strategies



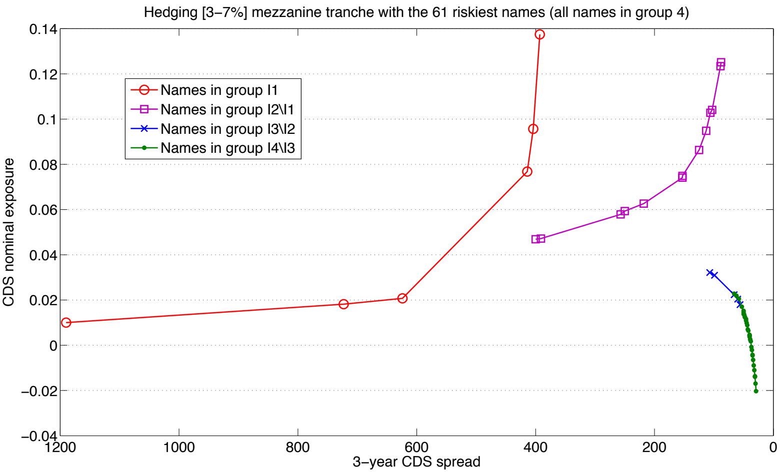
Min-variance hedging strategies

Hedging [0-3%] equity tranche with the 61 riskiest CDS-s (all name in I_4)



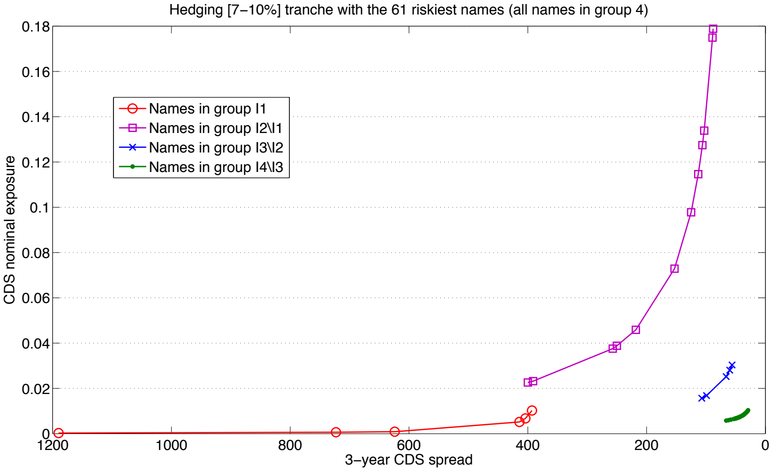
Min-variance hedging strategies

Hedging [3-7%] tranche with the 61 riskiest CDS-s (all name in I_4)

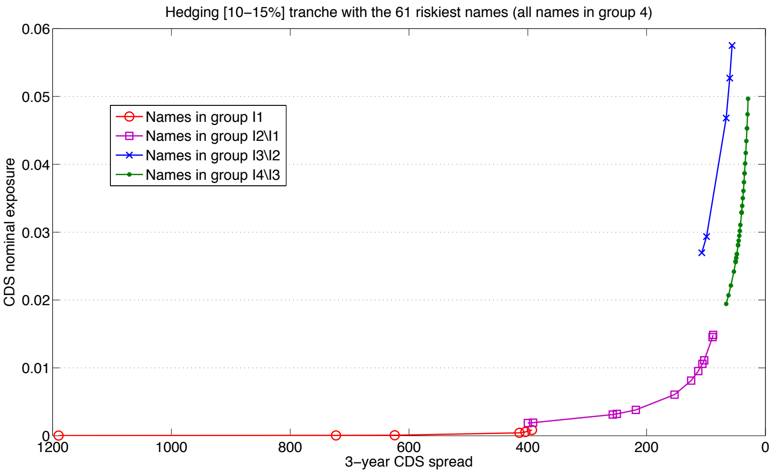


Min-variance hedging strategies

Hedging [7-10%] tranche with the 61 riskiest CDS-s (all name in I_4)

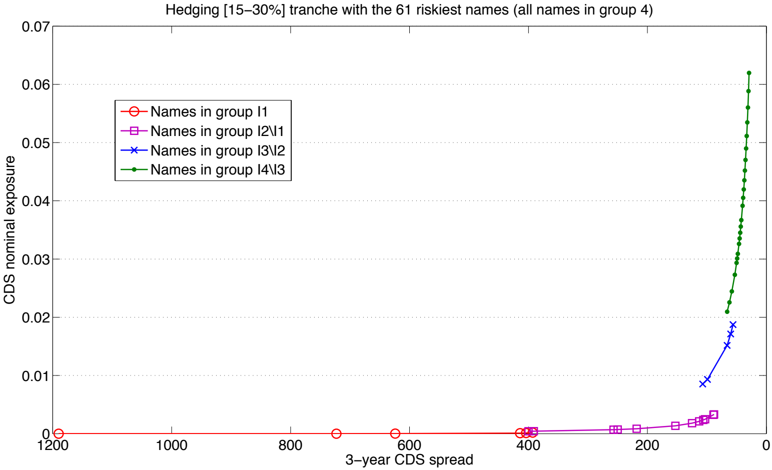


Hedging [10-15%] tranche with the 61 riskiest CDS-s (all name in I_4)



Min-variance hedging strategies

Hedging [15-30%] tranche with the 61 riskiest CDS-s (all name in I_4)



Conclusion

In this paper, we construct a dynamic bottom-up model of portfolio credit risk:

- Markov-copula construction of default times: two-steps calibration procedure of model parameters
- Common-shocks representation of default times conditionally on any given state of the Markov model: fast numerical computation of conditional loss distributions
- The model allows us to hedge CDO tranches using single-name CDS-s in a theoretically sound and practical convenient way

Thank you for your attention!

References

Bielecki, T.R., Vidozzi, A. and Vidozzi, L.: A Markov Copulae Approach to Pricing and Hedging of Credit Index Derivatives and Ratings Triggered Step–Up Bonds, *J. of Credit Risk*, 2008.

Brigo, D., Pallavicini, A., Torrential, R.: Calibration of CDO Tranches with the Dynamical Generalized-Poisson Loss Model. *Working Paper*, 2006.

Elouerkhaoui, Y.: Pricing and Hedging in a Dynamic Credit Model. *International Journal of Theoretical and Applied Finance*, Vol. 10, Issue 4, 703–731, 2007.

Lindskog, F. and McNeil, A. J.: Common Poisson Shock Models: Applications to Insurance and Credit Risk Modelling. *ASTIN Bulletin*, 33(2), 209-238, 2003

Markovian model of portfolio credit risk

Comment on Markov copula property

- The Markov copula property satisfied by the model is known as the *strong Markov copula property*. This property prohibits default contagion between individual credit names.
- A weaker form of the Markov copula property, where for every $i = 1, \ldots, n$, the process (X^i, H^i) is an $\mathcal{F}^i$-Markov but not-necessarily $\mathcal{F}$-Markov, has also been studied. Such weak Markov copula property allows for default contagion between individual credit names.