Dynamic Hedging of Portfolio Credit Risk in a Markov Copula Model

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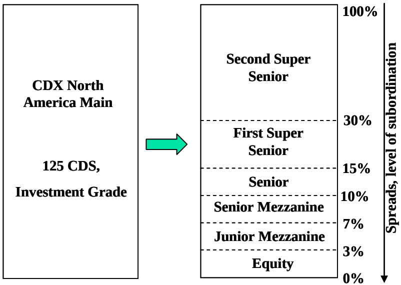
Introduction

Bielecki, Cousin, Crépey, Herbertsson (BCCH1)
Dynamic Modeling of Portfolio Credit Risk with Common-Shocks
to appear in *Journal of Optimization Theory and Application*

Bielecki, Cousin, Crépey, Herbertsson (BCCH2)
A Bottom-Up Dynamic Model of Portfolio Credit Risk.
Part I: Markov Copula Perspective
Part II: Common-Shock Interpretation, Calibration and Hedging Issues

Bielecki, Cousin, Crépey, Herbertsson (BCCH3)
A Bottom-Up Dynamic Model of Portfolio Credit Risk with Stochastic Intensities and Random Recoveries

Main issue: hedging of portfolio credit derivatives



- Cash-flows driven by the realized path of the aggregate loss process

$$L_t = \frac{1}{n}\sum_{i=1}^{n}(1 - R_i)H_t^i$$

where R_i is the recovery rate and H_t^i is the default indicator of obligor i

Introduction

In this paper, we construct a bottom-up dynamic model consisting of

- $\mathbf{X} = (X^1, \ldots, X^n)$ a vector of factor processes driving credit spreads
- $\mathbf{H} = (H^1, \ldots, H^n)$ a vector of default processes ($H_t^i = 1$ iif default of name i occurs before time t)
- $\mathcal{F}_t = \mathcal{F}_t^{\mathbf{X},\mathbf{H}}$

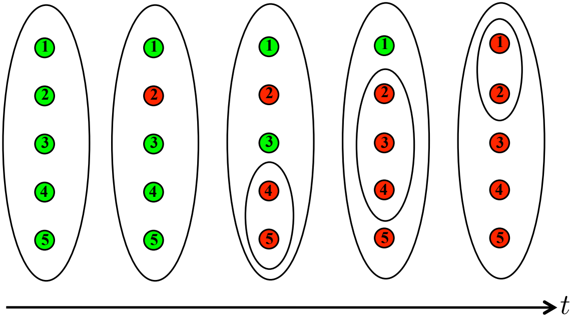
and with the following key features :

- Markovian model: $(\mathbf{X}, \mathbf{H})$ is a Markov process with respect to $\mathcal{F}$
- Markov copula property: Each pair (X^i, H^i) is a Markov process with respect to $\mathcal{F}$
- Tractable model: Computation of CDS spreads, CDO tranche prices and hedging strategies can be achieved by fast numerical procedure

Simultaneous default model

- Defaults are the consequence of triggering-events affecting pre-specified groups of obligors

Example: $n = 5$ and $\mathcal{Y} = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{4,5\}, \{2,3,4\}, \{1,2\}\}$.



Markov copula model of portfolio credit risk

- $\{1, \ldots, n\}$ set of credit references
- $\mathcal{Y} = \{\{1\}, \ldots, \{n\}, I_1, \ldots, I_m\}$ pre-specified groups of obligors
- $\lambda_Y = \lambda_Y(t)$ deterministic intensity function of the triggering-event associated with group $Y \in \mathcal{Y}$
- $\mathbf{H}_t = (H_t^1, \ldots, H_t^n)$ defined as an n-dimensional Markov chain in $\{0,1\}^n$ such that for $\mathbf{k}, \mathbf{m} \in \{0,1\}^n$:

$$\mathbb{P}\left(\mathbf{H}_{t+dt} = \mathbf{m} \mid \mathbf{H}_t = \mathbf{k}\right) = \sum_{Y \in \mathcal{Y}} \lambda_Y(t) \mathbf{1}_{\{\mathbf{k}^Y = \mathbf{m}\}} dt$$

  where $\mathbf{k}^Y$ is obtained from $\mathbf{k} = (k_1, \ldots, k_n)$ by replacing the components k_j, $j \in Y$, by number one. ex: $(0,1,0,0)^{\{1,2,4\}} = (1,1,0,1)$
- $\mathcal{F}_t = \sigma(\mathbf{H}_u\, , u \leq t)$ natural filtration of $\mathbf{H}$

Example: Portfolio with $n = 2$ names

$\mathcal{Y} = \{\{1\},\{2\},\{1,2\}\}$. $\mathbf{H}_t = (H_t^1, H_t^2)$ is a bivariate continuous-time Markov chain with space set $\{(0,0),(1,0),(0,1),(1,1)\}$ and generator matrix

$$
\begin{array}{c}
(0,0)\\ (1,0)\\ (0,1)\\ (1,1)
\end{array}
\overset{\begin{array}{cccc}(0,0) & (1,0) & (0,1) & (1,1)\end{array}}{
\begin{pmatrix}
- & \lambda_{\{1\}} & \lambda_{\{2\}} & \lambda_{\{1,2\}} \\
0 & - & 0 & \lambda_{\{2\}} + \lambda_{\{1,2\}} \\
0 & 0 & - & \lambda_{\{1\}} + \lambda_{\{1,2\}} \\
0 & 0 & 0 & 0
\end{pmatrix}}
$$

- Obligor 1 defaults with intensity $\lambda_{\{1\}} + \lambda_{\{1,2\}}$ regardless of the state of the pool
- Obligor 2 defaults with intensity $\lambda_{\{2\}} + \lambda_{\{1,2\}}$ regardless of the state of the pool

Markov copula model of portfolio credit risk

n-dimensional case

Obligor i defaults with intensity

$$\lambda_i(t) = \lambda_{\{i\}}(t) + \sum_{k=1}^{m} \lambda_{I_k}(t)\mathbf{1}_{\{i \in I_k\}}$$

Then,

$$\mathbb{P}(H^i_{t+dt} - H^i_t = 1 \mid \mathcal{F}_t) = \mathbb{P}(H^i_{t+dt} - H^i_t = 1 \mid H^i_t) = (1 - H^i_t)\lambda_i(t)dt$$

- Each default indicator process H^i, $i = 1, \ldots, n$ is Markov with respect to $\mathcal{F}$: strong Markov copula property (Bielecki, Vidozzi and Vidozzi 2008)
- On economic grounds, this means that there is no contagion effect : past defaults do not have any effect on intensities of surviving names

The previous construction can be extended to the case of stochastic intensity functions:

$$\lambda_Y = \lambda_Y(t, X_t^Y)\,,\ Y \in \mathcal{Y}$$

where $\mathbf{X} = (X^Y)_{Y \in \mathcal{Y}}$ is a strong solution to

$$dX_t^Y = b_Y(t, X_t^Y)\,dt + \sigma_Y(t, X_t^Y)\,dW_t^Y\,, \quad (1)$$

with

- $b_Y(t, x)$ and $\sigma_Y(t, x)$ are suitable drift and diffusion functions
- W^Y, $Y \in \mathcal{Y}$ are independent standard brownian motions

Markov copula model of portfolio credit risk

Markov property of the model

Let $\mathcal{F} = \mathcal{F}^{\mathbf{X},\mathbf{H}}$ be the natural filtration of $(\mathbf{X}, \mathbf{H})$. The process $(\mathbf{X}, \mathbf{H})$ is an $\mathcal{F}$-Markov process with generator $\mathcal{A}$ given by

$$\mathcal{A}_t u(t, \mathbf{x}, \mathbf{k}) = \sum_{Y \in \mathcal{Y}} \Big(b_Y(t, x_Y) \partial_{x_Y} u(t, \mathbf{x}, \mathbf{k}) + \frac{1}{2} \sigma_Y^2(t, x_Y) \partial^2_{x_Y^2} u(t, \mathbf{x}, \mathbf{k}) \Big) + \sum_{Y \in \mathcal{Y}} \lambda_Y(t, x_Y) \delta u^Y(t, \mathbf{x}, \mathbf{k})$$

Could we find $\mathbf{X}$ and intensity functions $\lambda_Y(\cdot, \cdot)$, $Y \in \mathcal{Y}$ such that the Markov copula property holds ?

- i.e., for every $i = 1, \ldots, n$, the process (X^i, H^i) is an $\mathcal{F}$-Markov process ?

Possible solution : generalized CIR intensities

$$\lambda_Y(t, \mathbf{X}_t) = X_t^Y \ , \ Y \in \mathcal{Y}$$

where X_t^Y, $Y \in \mathcal{Y}$ are generalized CIR processes such that

$$dX_t^Y = a\left(b_Y(t) - X_t^Y\right)dt + c\sqrt{X_t^Y}\,dW_t^Y$$

- a and c are positive parameters common to all groups
- $b_Y(t)$ is specific to group Y and defined by a deterministic function of time
- W^Y, $Y \in \mathcal{Y}$ are independent brownian motions

Markov copula model of portfolio credit risk

Possible solution : generalized CIR intensities (cont.)

In this framework, the default intensity of name i is given by

$$X_t^i = X_t^{\{i\}} + \sum_{k=1}^{m} X_t^{I_k} \mathbf{1}_{\{i \in I_k\}}$$

Note that X^i is again a generalized CIR process such that

$$dX_t^i = a\left(b_i(t) - X_t^i\right) dt + c\sqrt{X_t^i} dW_t^i$$

where

- $b_i(t) = b_{\{i\}}(t) + \sum_{k=1}^{m} b_{I_k}(t) \mathbf{1}_{\{i \in I_k\}}$
- W^i is a standard brownian motion

Markov copula model of portfolio credit risk

Markov copula property for extended CIR intensities (Bielecki et al. (2012a))

Let $\mathcal{F} = \mathcal{F}^{\mathbf{X},\mathbf{H}}$ be the natural filtration of $(\mathbf{X}, \mathbf{H})$ where $\mathbf{X} = (X^Y)_{Y \in \mathcal{Y}}$ are previously defined CIR processes. Then,

- $(\mathbf{X}, \mathbf{H})$ is an $\mathcal{F}$-Markov process
- For every $i = 1, \ldots, n$, the process (X^i, H^i) is an $\mathcal{F}$-Markov process

Moreover, the $\mathcal{F}_t$–conditional survival probability of name i is given by

$$\mathbb{P}(\tau_i > T \,|\, \mathcal{F}_t) = (1 - H^i_t)\mathbb{E}\left\{\exp\left(-\int_t^T X^i_u du\right) \,|\, X^i_t\right\}$$

- Analytical expression available in the case of piecewise-constant $b^Y(\cdot)$ (See BCCH3)
- Other stable-by-convolution processes can be considered (e.g., processes driving by Levy-subordinators)

Common-Shock Model Interpretation

Main result: equivalent $\mathcal{F}_t$–related common-shocks model

- $\mathcal{Y}_t = \{Y \in \mathcal{Y}\,; Y \nsubseteq \text{supp}(\mathbf{H}_t)\}$: set of pre-specified groups that contain at least one alive obligor
- For any pre-specified group $Y \in \mathcal{Y}_t$, we define

$$\tau_Y(t) = \inf\left\{\theta \geq t \mid \int_t^\theta \lambda_Y(s, \mathbf{X}_s)ds > E_Y\right\}$$

  where E_Y, $Y \in \mathcal{Y}_t$, are independent and exponentially distributed random variables with parameter 1.
- In the $\mathcal{F}_t$–related common-shock model, the individual default time of a non-defaulted name i is defined by:

$$\widehat{\tau}_i(t) = \min_{\{Y \in \mathcal{Y}_t\,;\, i \in Y\}} \tau_Y(t)$$

- $H^i_\theta(t) = \mathbf{1}_{\{\widehat{\tau}_i(t) \leq \theta\}}$: default indicator of name i at time θ in the $\mathcal{F}_t$–related common-shock model

Main result

Proposition

Let Z be a subset of $\{1, \ldots, n\}$. For every $\theta_1, \ldots, \theta_n \geq t$, one has on the event $\{Z = \text{supp}^c(\mathbf{H}_t)\}$:

$$\mathbb{P}\left(\tau_i > \theta_i,\, i \in \text{supp}^c(\mathbf{H}_t) \mid \mathcal{F}_t\right) = \mathbb{P}\left(\widehat{\tau}_i(t) > \theta_i,\, i \in Z \mid \mathbf{X}_t\right)$$

Moreover, if

- $N_\theta = \sum_{i=1}^n H_\theta^i$ denotes the cumulative number of defaults at time θ in the Markov model
- $N_\theta(t, Z) = n - |Z| + \sum_{i \in Z}^n H_\theta^i(t)$ denotes the cumulative number of defaults at time θ in the $\mathcal{F}_t$-related common-shock model

then, for every $\theta \geq t$, one has on the event $\{Z = \text{supp}^c(\mathbf{H}_t)\}$:

$$\mathbb{P}\left(N_\theta = k \mid \mathcal{F}_t\right) = \mathbb{P}\left(N_\theta(t, Z) = k \mid \mathbf{X}_t\right)$$

for any $k = n - |Z|, \ldots, n$.

Common-Shock Model Interpretation

- But, for any time $\theta \geq t$, $H_\theta^i(t)$, $i \in Z$, are conditionally independent Bernoulli's given $\left(H_\theta^{I_1}(t), \ldots, H_\theta^{I_m}(t)\right)$
- Fast convolution-recursion procedure can be used to compute marginal loss distributions conditionally on any given set $\{Z = \text{supp}^c(\mathbf{H}_t)\}$.
- As far as standard CDO tranches are concerned, we will see that pricing, calibration and computation of hedging strategies are numerically tractable

Hedging CDO tranches with single-name CDS-s

Set of fundamental martingales for jump components of the Markov model

- H_t^Z is the indicator process of simultaneous default of names in the set Z, for every subset Z of $\{1, \ldots, n\}$
- $Y_t = Y \cap \text{supp}^c(\mathbf{H}_{t-})$ stands for the set of survivors of set Y right before t, for every pre-specified group $Y \in \mathcal{Y}$

Set of fundamental martingales

The process M^Z defined by

$$dM_t^Z := dH_t^Z - \ell_Z(t, \mathbf{X}_t, \mathbf{H}_{t-})dt$$

is a martingale with respect to $\mathcal{F}$, where the intensity function $\ell_Z(t, \mathbf{x}, \mathbf{k})$ is such that

$$\ell_Z(t, \mathbf{X}_t, \mathbf{H}_{t-}) = \sum_{Y \in \mathcal{Y};\, Y_t = Z} \lambda_Y(t, \mathbf{X}_t)$$

Hedging CDO tranches with single-name CDS-s

Itô formula

Given a "regular enough" function $u = u(t, \mathbf{x}, \mathbf{k})$, one has, for $t \in [0, T]$,

$$du(t, \mathbf{X}_t, \mathbf{H}_t) = \Big(\partial_t + \mathcal{A}_t\Big) u(t, \mathbf{X}_t, \mathbf{H}_t) dt \, + \nabla u(t, \mathbf{X}_t, \mathbf{H}_t) \sigma(t, \mathbf{X}_t) d\mathbf{W}_t + \sum_{Z \in \mathcal{Z}_t} \delta u^Z(t, \mathbf{X}_t, \mathbf{H}_{t-}) dM_t^Z$$

where

- $\sigma(t, \mathbf{x})$: diagonal matrix with diagonal $(\sigma_Y(t, x_Y))_{Y \in \mathcal{Y}}$
- $\nabla u(t, \mathbf{x}, \mathbf{k}) = (\partial_{x_1} u(t, \mathbf{x}, \mathbf{k}), \ldots, \partial_{x_\nu} u(t, \mathbf{x}, \mathbf{k}))$
- $\delta u^Z(t, \mathbf{x}, \mathbf{k}) = u(t, \mathbf{x}, \mathbf{k}^Z) - u(t, \mathbf{x}, \mathbf{k})$
- $\mathcal{Z}_t = \{Y_t;\, Y \in \mathcal{Y}\} \setminus \emptyset$: set of all non-empty sets of survivors of sets Y in $\mathcal{Y}$ right before time t

Martingale dimension: $n + m + 2^n$

Hedging CDO tranches with single-name CDS-s

Price dynamics for single-name CDS-s (buy-protection position)

- T: contract maturity
- S_i: T-year contractual CDS-spread of obligor i
- $t_1 < \cdots < t_p = T$: premium payment dates, $h = t_j - t_{j-1}$ length between two premium payment dates (typically a quarter)
- R_i: recovery rate of obligor i

Except for numerical results, we will assume zero interest rates

Hedging CDO tranches with single-name CDS-s

Price dynamics for single-name CDS i

The price P^i and the cumulative value $\widehat{P}^i$ at time $t \in [0, T]$ of a single-name CDS on obligor i are given by

$$\begin{aligned} P^i_t &= \mathbf{1}_{\{\tau_i > t\}} v_i(t, X^i_t) \\ d\widehat{P}^i_t &= \mathbf{1}_{\{\tau_i > t\}} \partial_{x_i} v_i(t, X^i_t) \sigma_i(t, X^i_t) dW^i_t \\ &\quad + \sum_{Z \in \mathcal{Z}_t} \mathbf{1}_{\{i \in Z\}} \Big(1 - R_i - v_i(t, X^i_t)\Big) \, dM^Z_t \end{aligned}$$

for a pre-default pricing function $v_i(t, x_i)$ such that

$$\mathbf{1}_{\{\tau_i > t\}} v_i(t, X^i_t) = \mathbb{E}[(1 - R_i) \mathbf{1}_{\{t < \tau_i \le T\}} - S_i h \sum_{t < t_j \le T} \mathbf{1}_{\{\tau_i > t_j\}} | \mathcal{F}_t]$$

Price dynamics for CDO tranche $[a, b]$ (buy-protection position)

- T: contract maturity
- a: attachement point, b: detachement point, $0 \leq a < b \leq 1$
- $S^{a,b}$: T-year contractual spread of CDO tranche $[a, b]$
- $t_1 < \cdots < t_p = T$: premium payment dates, $h = t_j - t_{j-1}$ length between two premium payment dates (typically a quarter)
- CDO tranche cash-flows are driven by the tranche loss process

$$L_t^{a,b} = L_{a,b}(\mathbf{H}_t) = (L_t - a)^+ - (L_t - b)^+$$

where

$$L_t = L_t(\mathbf{H}_t) = \frac{1}{n}\sum_{i=1}^{n}(1 - R_i)H_t^i$$

is the credit loss process for the underlying portfolio

Hedging CDO tranches with single-name CDS-s

Price dynamics for CDO tranche $[a, b]$

The price Π and the cumulative value $\widehat{\Pi}$ at time $t \in [0, T]$ of a CDO-tranche $[a, b]$ are given by

$$\begin{aligned}
\Pi_t &= u(t, \mathbf{X}_t, \mathbf{H}_t) \\
d\widehat{\Pi}_t &= \nabla u(t, \mathbf{X}_t, \mathbf{H}_t)\sigma(t, \mathbf{X}_t)d\mathbf{W}_t \\
&\quad + \sum_{Z \in \mathcal{Z}_t} \Big(L_{a,b}(\mathbf{H}^Z_{t-}) - L_{a,b}(\mathbf{H}_{t-}) + \delta u^Z(t, \mathbf{X}_t, \mathbf{H}_{t-}) \Big)\, dM^Z_t
\end{aligned}$$

for a pricing function $u(t, \mathbf{x}, \mathbf{k})$ such that

$$u(t, \mathbf{X}_t, \mathbf{H}_t) = \mathbb{E}\Big[L^{a,b}_T - L^{a,b}_t - S^{a,b}\, h \sum_{t < t_j \leq T} \Big(b - a - L^{a,b}_{t_j} \Big) \,\Big|\, \mathcal{F}_t \Big]$$

The pricing function $u(t, \mathrm{x}, \mathrm{k})$ solves a very large system of Kolmogorov pde. Thanks to the common-shock interpretation, it can be computed by fast recursion procedures.

Hedging portfolio: first d single-name CDS-s and the savings account

The vector of cumulative values $\widehat{\mathbf{P}} = (\widehat{P}^1, \dots, \widehat{P}^d)^\mathsf{T}$ associated with the first d CDS-s has the following dynamics:

$$d\widehat{\mathbf{P}}_t = \nabla \mathbf{v}(t, \mathbf{X}_t, \mathbf{H}_t)\sigma(t, \mathbf{X}_t)d\mathbf{W}_t + \sum_{Z \in \mathcal{Z}_t} \Delta \mathbf{v}^Z(t, \mathbf{X}_t, \mathbf{H}_{t-})dM_t^Z$$

where

- $\nabla \mathbf{v}$ is a $d \times \nu$-matrix such that $\nabla \mathbf{v}(t, \mathbf{x}, \mathbf{k})_i^j = \mathbf{1}_{\{k_j=0\}} \partial_{x_j} v_i(t, x_i)$, for every $1 \leq i \leq d$ and $1 \leq j \leq \nu$
- $\Delta \mathbf{v}^Z(t, \mathbf{x}, \mathbf{k})$ is a d-dimensional column vector equal to

$$\left(\mathbf{1}_{\{1 \in Z,\, k_1=0\}} \left((1 - R_1) - v_1(t, x_1)\right), \dots, \mathbf{1}_{\{d \in Z,\, k_d=0\}} \left((1 - R_d) - v_d(t, x_d)\right)\right)^\mathsf{T}$$

Hedging CDO tranches with single-name CDS-s

Tracking error: Process (e_t) such that $e_0 = 0$ and for $t \in [0, T]$:

$$
\begin{aligned}
de_t &= d\widehat{\Pi}_t - \zeta_t d\widehat{\mathbf{P}}_t \\
&= \Big(\nabla u(t, \mathbf{X}_t, \mathbf{H}_t) - \zeta_t \nabla \mathbf{v}(t, \mathbf{X}_t, \mathbf{H}_t)\Big)\sigma(t, \mathbf{X}_t) d\mathbf{W}_t \\
&\quad + \sum_{Z \in \mathcal{Z}_t} \Big(\Delta u^Z(t, \mathbf{X}_t, \mathbf{H}_{t-}) - \zeta_t \Delta \mathbf{v}^Z(t, \mathbf{X}_t, \mathbf{H}_{t-})\Big) dM_t^Z
\end{aligned}
$$

where

- $\zeta_t = (\zeta_t^1, \ldots, \zeta_t^d)$ gives the positions held at time t in CDS $1, \ldots, d$
- $\nabla u(t, \mathbf{x}, \mathbf{k}) = (\partial_{x_1} u(t, \mathbf{x}, \mathbf{k}), \ldots, \partial_{x_\nu} u(t, \mathbf{x}, \mathbf{k}))$
- $\Delta u^Z(t, \mathbf{x}, \mathbf{k}) = \delta^Z u(t, \mathbf{x}, \mathbf{k}) + L_{a,b}(\mathbf{k}^Z) - L_{a,b}(\mathbf{k})$

Hedging CDO tranches with single-name CDS-s

Min-variance hedging strategies

The min-variance hedging strategy ζ for the CDO-tranche $[a, b]$ is

$$\zeta_t = \frac{d\langle \widehat{\Pi}, \widehat{\mathbf{P}} \rangle_t}{dt} \left(\frac{d\langle \widehat{\mathbf{P}} \rangle_t}{dt} \right)^{-1} = \zeta(t, \mathbf{X}_t, \mathbf{H}_{t-})$$

where $\zeta = (u, \mathbf{v})(\mathbf{v}, \mathbf{v})^{-1}$, with

$$(u, \mathbf{v}) = (\nabla u)\sigma^2 (\nabla \mathbf{v})^{\mathsf{T}} + \sum_{Y \in \mathcal{Y}} \lambda_Y \Delta u^Y (\Delta \mathbf{v}^Y)^{\mathsf{T}}$$

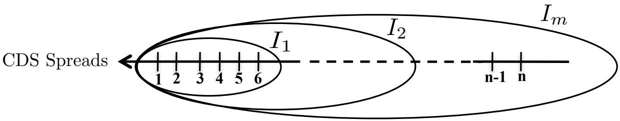
$$(\mathbf{v}, \mathbf{v}) = (\nabla \mathbf{v})\sigma^2 (\nabla \mathbf{v})^{\mathsf{T}} + \sum_{Y \in \mathcal{Y}} \lambda_Y \Delta \mathbf{v}^Y (\Delta \mathbf{v}^Y)^{\mathsf{T}}$$

Data set: 5-year CDX North-America IG index on 20 December 2007

- Quoted spreads at different pillars of the $n = 125$ index constituents
- Quoted spreads of standard tranches [0,3], [3,7], [7,10], [10,15], [15,30]

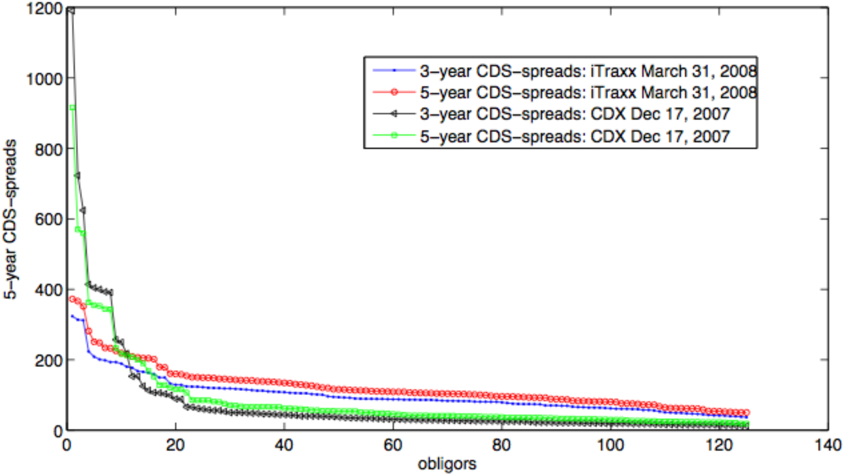
Model specification:

- Names are labelled with respect to decreasing level of spreads



- $m = 5$ groups $I_1 \subset \cdots \subset I_5$ such that $I_1 = \{1, \ldots, 6\}$, $I_2 = \{1, \ldots, 19\}$, $I_3 = \{1, \ldots, 25\}$, $I_4 = \{1, \ldots, 61\}$, $I_5 = \{1, \ldots, 125\}$
- Piecewise-constant intensities $\lambda_{\{1\}}, \ldots, \lambda_{\{125\}}$, $\lambda_{I_1}, \ldots, \lambda_{I_5}$ with grid points corresponding to CDS pillars
- Homogeneous and constant recovery rates: 40%
- Constant short-term interest rate: 3%

Calibration on CDX index



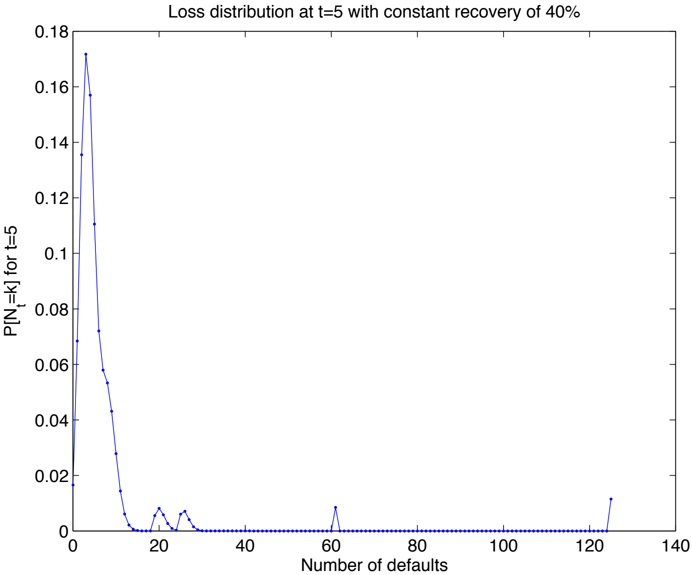
Calibration on CDX index

Calibration results:

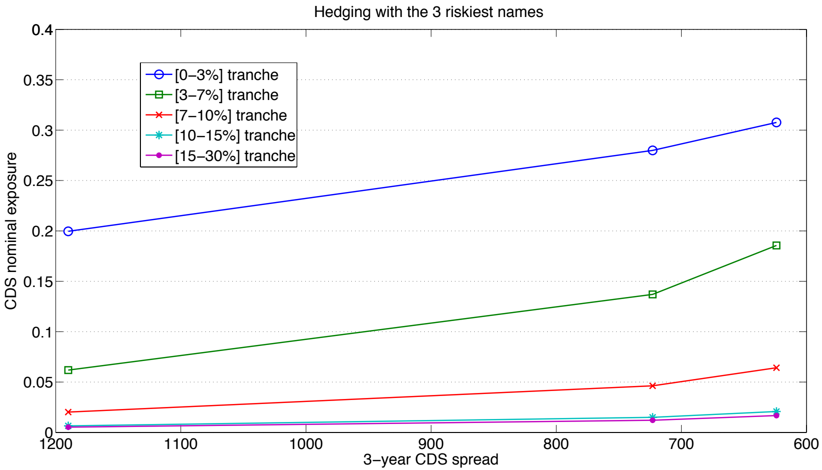
Tranche	[0,3]	[3,7]	[7,10]	[10,15]	[15,30]
Model spread in bps	48.0701	254.0000	124.0000	61.0000	38.9390
Market spread in bps	48.0700	254.0000	124.0000	61.0000	41.0000
Abs. Err. in bps	0.0001	0.0000	0.0000	0.0000	2.0610
% Rel. Err.	0.0001	0.0000	0.0000	0.0000	5.0269

- Names in the set $I_5 \setminus I_4$ are excluded from the calibration constraints (they can only default within the Armageddon shock I_5)

Implied 5-year loss distribution:



Min-variance hedging strategies



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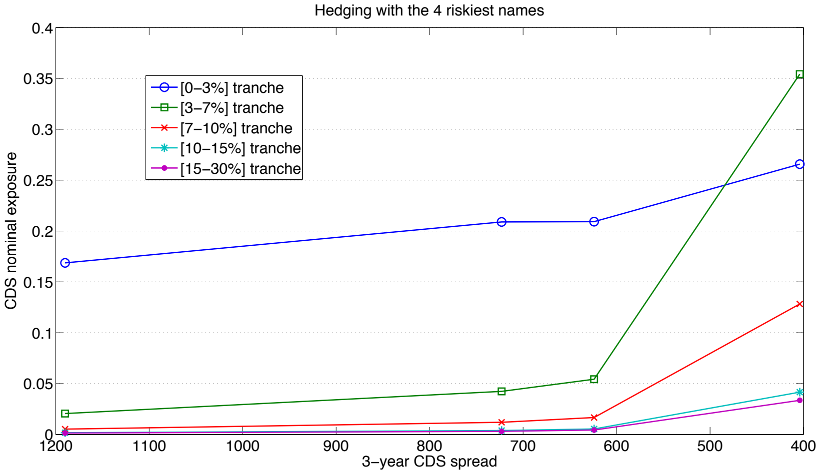

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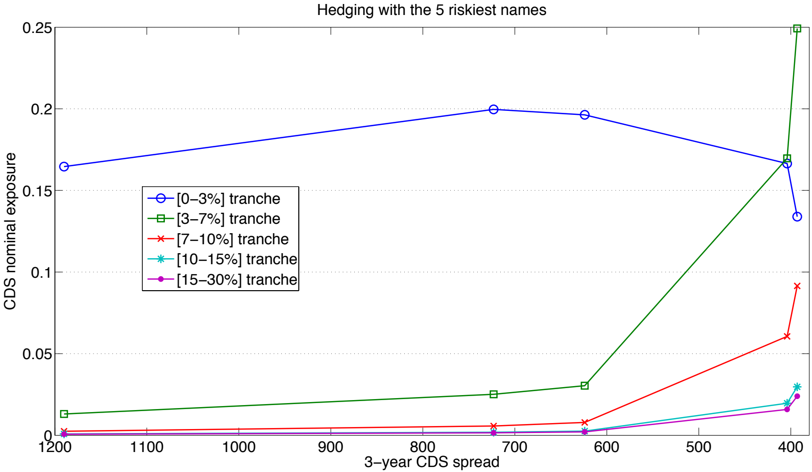
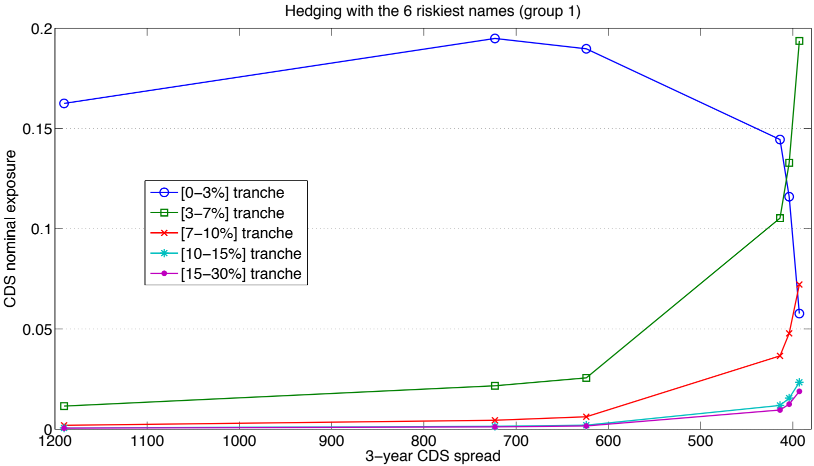


Min-variance hedging strategies



Conclusion

In this paper, we construct a dynamic bottom-up model of portfolio credit risk:

- Markov-copula construction of default times: two-steps calibration procedure of model parameters
- Common-shocks representation of default times conditionally on any given state of the Markov model: fast numerical computation of conditional loss distributions
- The model allows us to hedge CDO tranches using single-name CDS-s in a theoretically sound and practical convenient way

Conclusion

Thank you for your attention!

References

Bielecki, T.R., Vidozzi, A. and Vidozzi, L.: A Markov Copulae Approach to Pricing and Hedging of Credit Index Derivatives and Ratings Triggered Step–Up Bonds, *J. of Credit Risk*, 2008.

Brigo, D., Pallavicini, A., Torrential, R.: Calibration of CDO Tranches with the Dynamical Generalized-Poisson Loss Model. *Working Paper*, 2006.

Elouerkhaoui, Y.: Pricing and Hedging in a Dynamic Credit Model. *International Journal of Theoretical and Applied Finance*, Vol. 10, Issue 4, 703–731, 2007.

Lindskog, F. and McNeil, A. J.: Common Poisson Shock Models: Applications to Insurance and Credit Risk Modelling. *ASTIN Bulletin*, 33(2), 209-238, 2003

Comment on Markov copula property

- The Markov copula property satisfied by the model is known as the *strong Markov copula property*. This property prohibits default contagion between individual credit names.
- A weaker form of the Markov copula property, where for every $i = 1, \dots, n$, the process (X^i, H^i) is an $\mathcal{F}^i$-Markov but not-necessarily $\mathcal{F}$-Markov, has also been studied. Such weak Markov copula property allows for default contagion between individual credit names.